Chapter 7

Costs and Cost Minimization

Solutions to Problems

7.1 a) $500

b) 30% of $500, or $150

c) By not lowering the price and assuming the firm cannot sell any more printers, the best the firm can hope for is the $150 the firm can receive from the manufacturer. If the firm drops the price to $200 and sells the printers on their own they can actually "profit" an additional $50 over their best available alternative.

7.2 The accounting costs are simply the sum: 25,000 + 75,000 + 80,000 + 6,000 = $186,000 and the shop's accounting profit is $64,000 which means that Mr. Moore's total gain from this venture is 80,000 + 64,000 = $144,000.

The economic costs also include the opportunity cost of the land rental ($100,000) and of Mr. Moore's next best alternative, which in this case is $95,000. That is, Mr. Moore loses $15,000 by not choosing his next best alternative. Therefore Mr. Moore's total economic costs are 186,000 + 100,000 + 15,000 = $301,000, which exceeds his revenues by $51,000.

If he were to shut down the shop, Mr. Moore would earn 100,000 + 95,000 = $195,000 which is more than the $144,000 he currently earns (by precisely the $51,000 figure from above). Therefore he should shut down the shop.

7.3 At the optimum we must have

$$\frac{MP_K}{r} = \frac{MP_L}{w}$$

In this problem we have

$$\frac{200}{0.25} > \frac{1000}{10}$$
$$800 > 100$$

This implies that the firm receives more output per dollar spent on an additional machine hour of fermentation capacity than for an additional hour spent on labor. Therefore, the

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firm could lower cost while achieving the same level of output by using fewer hours of labor and more hours of fermentation capacity.

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7.4 a) If the price of both inputs change by the same percentage amount, the slope of the isocost line will not change. Since we are holding the level of output fixed, the isocost line will be tangent to the isoquant at the same point as prior to the price increase. Therefore, the cost-minimizing quantities of the inputs will not change.

b) If the price of capital increases by a larger percentage than the price of labor, then, relatively speaking, the price of labor has become cheaper. The firm will substitute away from capital and add labor until either the tangency condition holds or a corner solution is reached.

7.5 a) The amount of land used in production is fixed in the short-run. Hence, in the short-run the farmer chooses amount of capital and labor. It follows that cost-minimizing quantities of labor and capital have to satisfy equation MP_L / MP_K = w/r where w and r denote prices of labor and capital. Notice that w/r = (1.05 w)/(1.05 r). The cost-minimizing quantities of inputs, for each level of output, do not change when prices of both inputs go up by 5% and quantity of land is fixed.

b) For a given output level, the cost-minimizing farmer uses more capital and less labor.

7.6 Imagine that two expansion paths did cross at some point. Recall that the expansion path traces out the cost- minimizing combinations of inputs as output increases. Essentially the expansion path traces out all of the tangencies between the isocost lines and isoquants. These tangencies occur at the point where

$$\frac{MP_L}{MP_K} = \frac{w}{r}$$

If the expansion paths cross at some point then the cost minimizing combination of inputs must be identical with both sets of prices. This would require that

$$\frac{MP_K}{r_1} = \frac{MP_L}{w_1} \text{ and } \frac{MP_K}{r_2} = \frac{MP_L}{w_2}$$

Unless the input prices are proportional, i.e. unless $w_1 / r_1 = w_2 / r_2$, it is not possible for both of these equations to hold. Therefore, it is not possible for the expansion paths to cross unless the prices are proportional, in which case the two expansion paths will be identical.

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7.7 The tangency condition implies

$$\frac{\left[L^{1/2}+K^{1/2}\right]K^{-1/2}}{r}=\frac{\left[L^{1/2}+K^{1/2}\right]L^{-1/2}}{w}$$
$$\frac{1}{r\sqrt{K}}=\frac{1}{w\sqrt{L}}$$
$$w\sqrt{L}=r\sqrt{K}$$
$$\frac{K}{L}=\frac{w^2}{r^2}$$

Given that $w=10$ and $r=1$, this implies

$$100=\frac{K}{L}$$
$$100L=K$$

Returning to the production function and assuming $Q=121,000$ yields

$$121,000=\left[L^{1/2}+K^{1/2}\right]^2$$
$$121,000=\left[L^{1/2}+(100L)^{1/2}\right]^2$$
$$121,000=\left[L^{1/2}+10L^{1/2}\right]^2$$
$$121,000=\left[11L^{1/2}\right]^2$$
$$121,000=121L$$
$$1,000=L$$

Since $K=100L$, $K=100(1000)=100,000$. The cost minimizing quantities of capital and labor to produce 121,000 airframes is $K=100,000$ and $L=1,000$.

7.8 The tangency condition implies

$$10=\frac{K}{L}$$
$$10L=K$$

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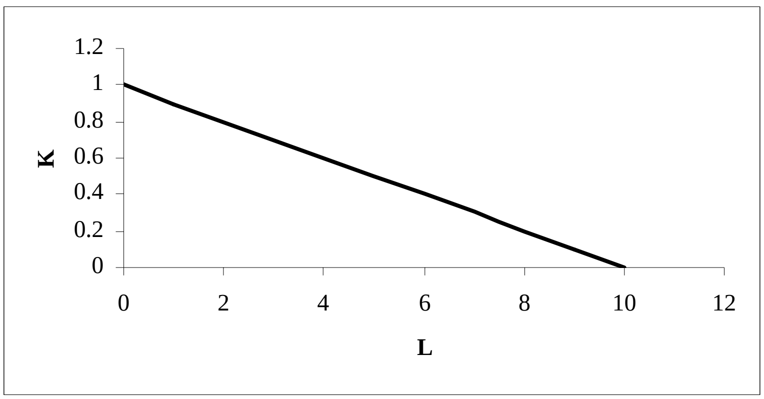
Substituting into the production function yields

$$121,000 = LK$$
$$121,000 = L(10L)$$
$$121,000 = 10L^2$$
$$12,100 = L^2$$
$$110 = L$$

Since $K = 10L$, $K = 1,100$. The cost-minimizing quantities of labor and capital to produce 121,000 airframes are $K = 1,100$ and $L = 110$.

7.9 a)



K and *L* are perfect substitutes, meaning that the production function is linear and the isoquants are straight lines. We can write the production function as $Q = 10,000K + 1000L$, where Q is the number of workers for whom payroll is processed.

b) If $r = 5$ and $w = 7.50$, the slope of a typical isocost line will be $-7.5/5.0 = -1.5$. This is steeper than the isoquant implying that the firm will employ only computer time (K) to minimize cost. The cost minimizing combination is $K = 1$ and $L = 0$. This outcome can be seen in the graph below. The isocost lines are the dashed lines.

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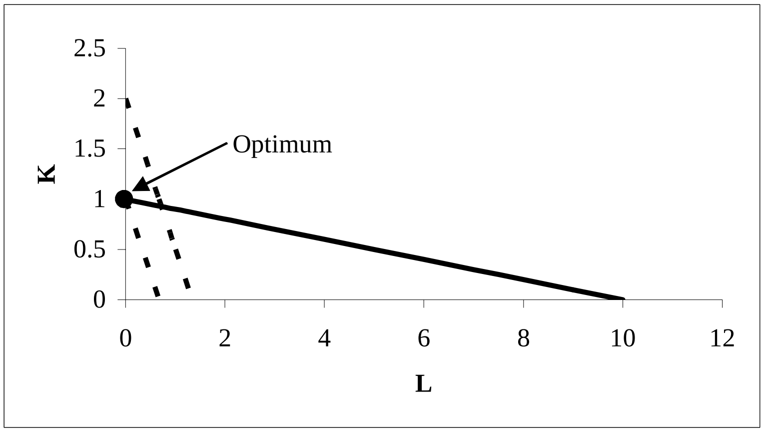

The total cost to process the payroll for 10,000 workers will be
$TC = 5(1) + 7.5(0) = 5$.

c) The firm will employ clerical time only if $MP_L / w > MP_K / r$. Thus we need 0.1 / 7.5 > 1/r or $r > 75$.

7.10 Currently the firm must be using $L = Q/K = 32/16 = 2$ units of labor. Let the factor prices of capital and labor be, respectively, r and w.
Its total expenditure is $C = wL + rK = 2(2) + 4(16) = 68$.
If it were to minimize cost, it would hire L and K so that
(1) $MP_K/r = MP_L/w$, or $L/4 = K/2$, or $L = 2K$ and (2) $Q = LK$.
(1) and (2) imply that $Q = 2K^2$, or $32 = 2K^2$, and thus $K = 4$ and $L = 8$.
So $Q = 32$ can be produced efficiently with a cost of $C = wL + rK = 2(8) + 4(4) = 32$.
The firm could save 68 – 32 = 36 by producing efficiently.

7.11 From the tangency condition, we get

$$\frac{K}{L} = \frac{w}{r}$$

$$K = \left(\frac{w}{r}\right)L$$

Substituting into the production function yields

$$Q = LK$$
$$Q = L\left(\frac{w}{r}\right)L$$
$$Q = \left(\frac{w}{r}\right)L^2$$
$$L = \left(\frac{rQ}{w}\right)^{1/2}$$

This represents the input demand curve for L. Since

$$K = \left(\frac{w}{r}\right)L$$

we have

$$K = \left(\frac{w}{r}\right)\left(\frac{rQ}{w}\right)^{1/2}$$
$$K = \left(\frac{wQ}{r}\right)^{1/2}$$

This represents the input demand curve for K.

7.12 Using the tangency condition, with the original input prices: $\frac{K}{L} = \frac{w}{r} = 2$. So, $K = 2L$. Also, using the information on total costs, $4L + 2K = 160$. Combining these two equations, we get (L, K) = (20, 40). Therefore the firm produces 20*40 = 800 units of output.

After the prices change, even though we don't know the numerical values of the input prices, we can still answer the question using the fact that we're told $w = 8r$. The tangency condition implies that $\frac{K}{L} = 8$, so $K = 8L$. Also, we have $KL = 800$. This implies that the optimal input combination is (L, K) = (10, 80).

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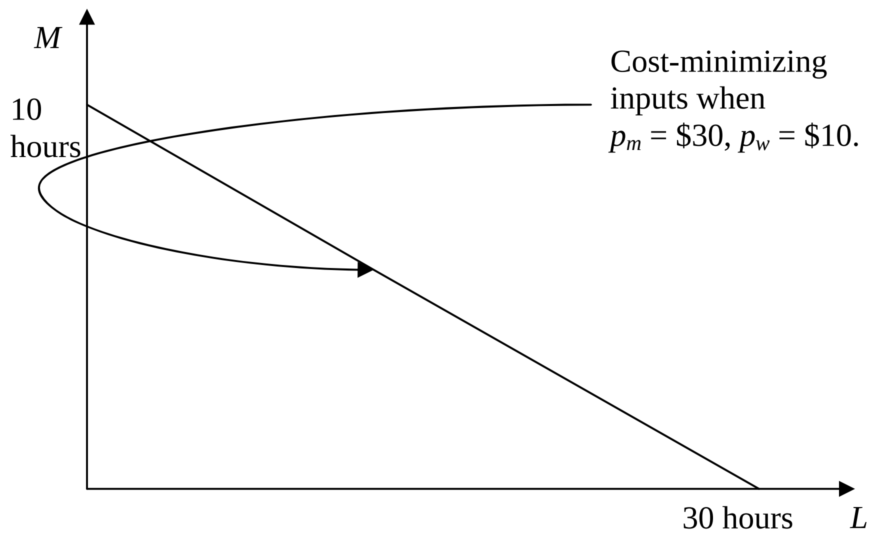
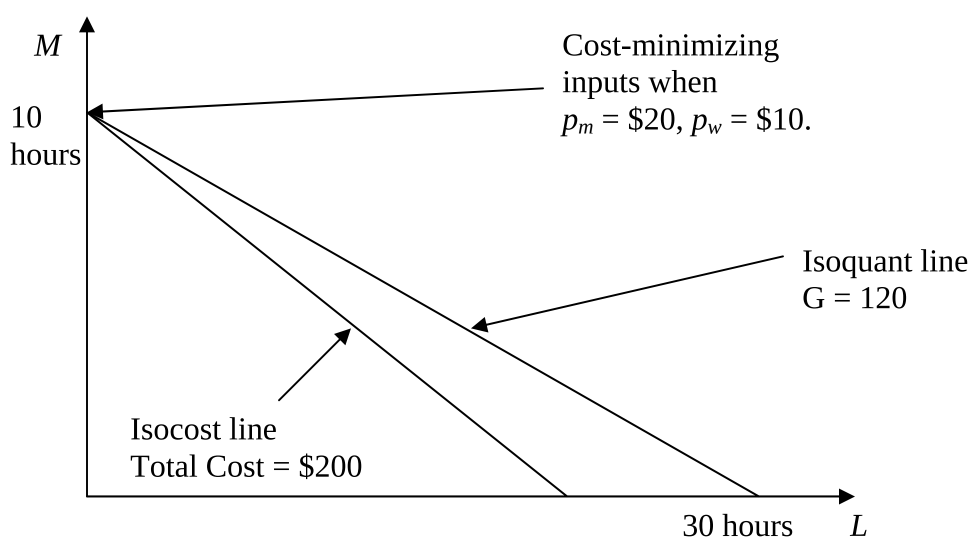
7.13 a) Isoquants for the production function are straight lines. At the given input prices slope of an isoquant is equal to the ratio of the input prices. Hence, all positive input quantities (measured in work hours) such that $4L + 12M = 120$ are cost-minimizing.



b) When one hour of the machine's work costs \$20 cost-minimizing firm does not use manual work at all. The cost-minimizing quantity of the machine's work necessary to produce 120 widgets is equal to $M = 120/12 = 10$ hours. The firm spends \$200. (Note that if the firm were to use only manual labor, the cost would be \$300 (= 30 hours x \$10 per hour).



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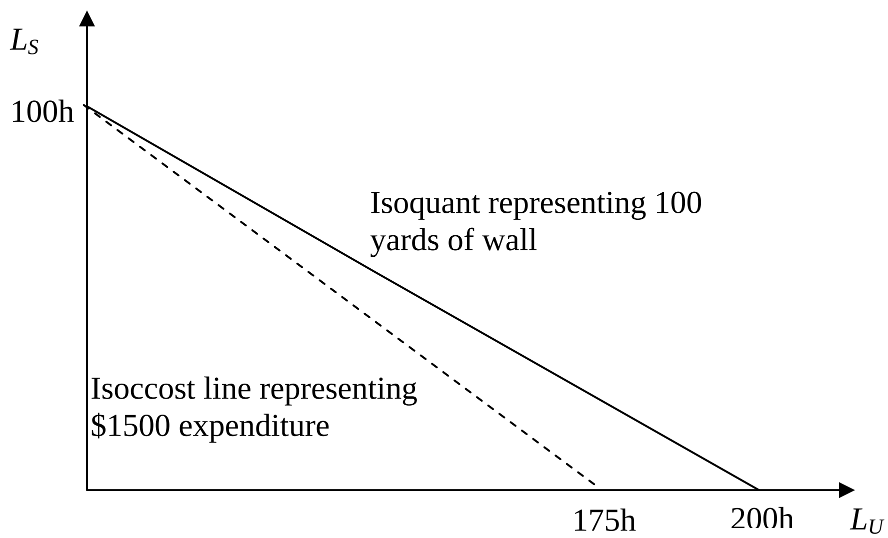
c) $G = 4L + 12M$

7.14 a) The production function is $Q = L_S + ½ L_U$ where L_S denotes hours worked by skilled workers and L_U denotes hours worked by unskilled workers. Both types of labor are perfect substitutes.

b) The isoquant is a straight line.



c) $MP_{Ls}/w_s = 1/15$; $MP_{Lu}/w_u = 0.5/8 = 1/16$. Thus, the "bang for the buck" is higher for skiled labor, and the firm will use only skilled labor.
Note that the total cost of building 100 yards with skilled labor is (100 hours)($15/ hour) = $1500.
The total cost of building 100 yards with unskilled labor is (200 hours)($8/ hour) = $1600.

The isocost line representing a $1500 expenditure is drawn as a dotted line in the graph in (b). The isocost line is more steeply sloped than the isoquant in the graph because the marginal rate of technical substitution of unskilled labor for unskilled labor is equal to ½, while the ratio of input prices is equal to 8/15.

7.15 a) First, note that this production function has diminishing $MRS_{L,K}$. The tangency condition would imply that $1/2\sqrt{L} = 1/50$ or $L = 625$. Substituting this back into the production function we see that $K = 10 – 25 = –15$. Since the firm cannot use a negative amount of capital, the tangency condition is not valid in this case.

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Looking at the corner with K = 0, since Q = 10 the firm requires $L = Q^2 = 100$ units of labor. At this point, MP_L / w = (1/20)/1 = 0.05 > MP_K / r = 1/50 = 0.02. Since the marginal product per dollar is higher for labor, the firm will use only labor and no capital.

b) The firm will use a positive amount of capital when $\frac{MP_L}{w} = \frac{MP_K}{r}$, or $2\sqrt{L} = r$. Thus $L = 0.25r^2$. From the production constraint $K = Q - \sqrt{L} = 10 - 0.5r$. So if $K > 0$ then we must have $10 - 0.5r > 0$, or $r < 20$.

c) Again, using the tangency condition we must have $2\sqrt{L} = r$. Therefore, since r = 50, L = 625. From the production constraint, the input demand for capital is $K = Q - \sqrt{L} = Q - 25$. So if $K > 0$ then we must have $Q > 25$.

7.16 No, these are not valid input demand curves. In both cases the quantity of the input is positively related to the input's price. Such upward-sloping input demand curves cannot exist.

7.17 If K = 0, then the firm must hire L = 5 units of labor. For this to be optimal, it must be that $MP_L / w > MP_K / r$, or $1/w > 6$. In other words, $w < 1/6$.

If L = 0, then the firm must hire K = 5 units of capital. For this to be optimal, it must be that $MP_L / w < MP_K / r$, or $6/w > 1$. In other words, $w > 6$.

For the firm to use both capital and labor, it must be that $1/6 < w < 6$. To see why, notice that the indifference curves will have diminishing $MRTS_{L,K}$. In particular, $MRTS_{L,K} = 6$ where the Q = 5 indifference curve intersects the K-axis (where L = 0). Diminishing $MRTS_{L,K}$ implies that the Q = 5 indifference curve will gradually flatten out until it intersects the L-axis (where K = 0), at which point $MRTS_{L,K} = 1/6$.

7.18 The input demand curves will be vertical lines, representing the fact that the demand by firms for such inputs is inelastic. If the firm's production function is $Q = \min(L, K)$ then, holding fixed the quantity of production and the price of capital, if the wage rate were to increase it would not change the firm's requirement for labor. Therefore, the demand for each input is independent of price and the demand curves are vertical lines.

7.19 Recall that with a linear production function we are usually going to get corner point solutions. In this case, the firm will employ only labor and no capital if labor is cheap enough or, $\frac{MP_L}{w} > \frac{MP_K}{r}$ i.e. if $w < \frac{br}{a}$. Similarly it will use just capital if the rental rate is low enough i.e. $r < \frac{aw}{b}$. If the firm uses only labor, it will use $L = \frac{Q}{a}$ units regardless of the price, and similarly it will use $K = \frac{Q}{b}$ units of capital if it uses any capital at all.

The input demand curve for labor for a given price, r, of capital, is shown below.

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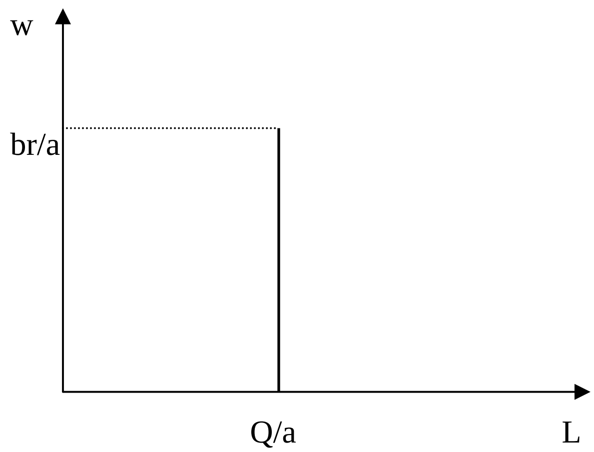
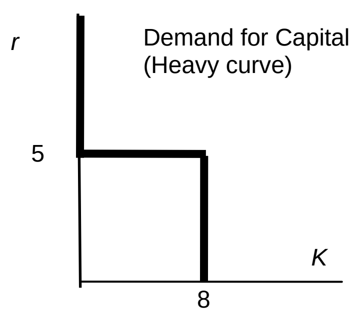


7.20 With this production function the firm views K and L as perfect substitutes. The firm will be at a corner point with $K = 0$ when $MP_K/r < MP_L/w$, or when $10/r < 2/1$, or when $r > 5$.

The firm will be at a corner point with $L = 0$ when $MP_K/r > MP_L/w$, or when $10/r > 2/1$, or when $r < 5$. When the firm needs to produce $Q = 80$, how much capital will it need? The production function shows that $80 = 10K$, or $K = 8$ units.

When r = 5, the firm might use any combination of K and L along the isoquant $80 = 10K + 2L$. The firm might therefore use any K such that $0 \leq K \leq 8$.

The graph of the demand for labor is as shown.



7.21 The tangency condition implies that $\frac{1}{2\sqrt{L}} = \frac{w}{r}$, or $\sqrt{L} = \frac{r}{2w}$. Clearly the demand curve for L is not a function of the level of output, Q. Therefore, as the level of output changes, the amount of labor is constant. Therefore, if we were to graph isoquants with labor on the horizontal axis, the expansion path for labor would just be a straight, vertical line.

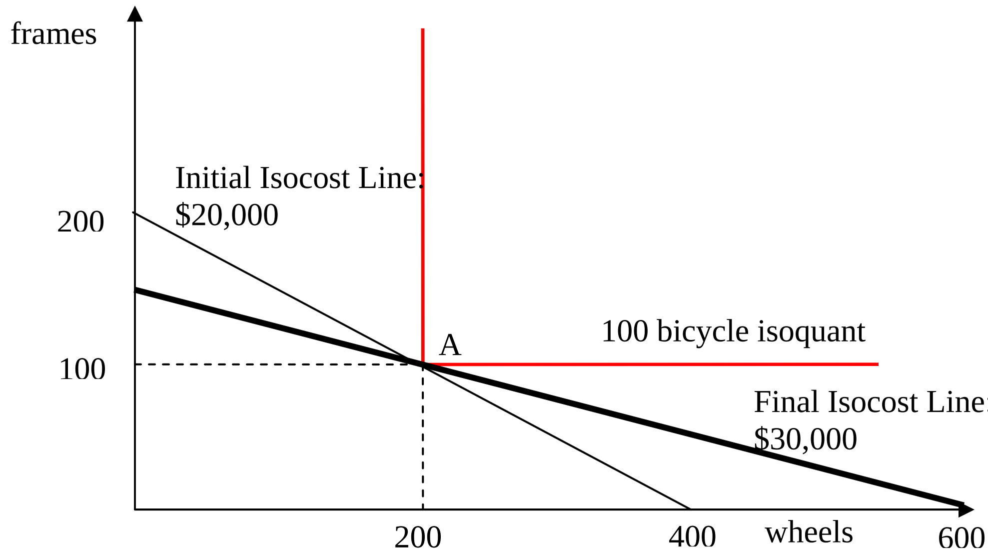
The demand curve for capital can be derived by substituting the demand curve for labor into the production function. That is, $K + \frac{r}{2w} = Q$, so $K = Q - \frac{r}{2w}$.

7.22 Using the tangency condition, initially $\frac{K}{L} = 1$, implying that $K = L$. Since $KL = 100$, we get $K = L = 10$.

Under the new prices, the tangency condition implies that $K=4L$. This means that the optimal input combination is $(L, K) = (5, 20)$.

The percent change in price is (4 – 1)*100 = 300%. While the percent change in the demand for labor is [(5 – 10)/10]*100 = –50%. Therefore the price elasticity of demand over this range of prices is –50/300 = –1/6.

7.23 a) The production function is $Q = \min(F, ½ W)$, where F denotes the number of frames and W denotes the number of wheels.



b) To produce 100 bicycles in the least costly manner, the firm always needs to choose basket A, with 200 wheels and 100 frames.
Initially, when the price of a frame is $100 and the price of a wheel is $50, the isocost line is the lighter one shown in the graph; all points on the isocost line indicate an expenditure of $20,000.
Later, when the price of a frame is $200 and the price of a wheel is $50, the isocost line is the lighter one shown in the graph; all points on the isocost line indicate an expenditure of $30,000.

7.24 With just two inputs, there is no tangency condition to worry about in the short run. To find the short-run cost-minimizing quantity of labor, we need only solve the production function for L in terms of Q and $\overline{K}$:

$$Q = 10\overline{K}L^{\frac{1}{3}}$$

This gives us:

$$L = \frac{Q^3}{1000\overline{K}^3}$$

This is the cost-minimizing quantity of labor in the short run.

7.25

a) Since $\overline{K} = 9$, we get $18L + 9 = 45$ which implies that L = 36/18 = 2. Therefore the firm's total cost with this input combination is 4(2) + 5(9) = $53.

b) If the firm could operate optimally, it would choose labor and capital to satisfy the tangency condition: $\frac{2K}{2L+1} = \frac{4}{5}$, implying that $10K = 8L + 4$. Also, $2KL + K = 45$. Combining these two conditions, $K = \sqrt{18}$ = 4.24 and L = 4.8. Now the firm's expenditure would be 4(4.24) + 5(4.8) = $41 approximately. Therefore the firm loses about $12 because of its constraint on capital.

7.26

a) Here we have two tangency conditions and the requirement that L, K, and M produce Q units of output.

$$\frac{MP_L}{MP_M} = \frac{w}{m} \Rightarrow \frac{\frac{1}{3}K^{\frac{1}{3}}L^{-\frac{2}{3}}M^{\frac{1}{3}}}{\frac{1}{3}K^{\frac{1}{3}}L^{\frac{1}{3}}M^{-\frac{2}{3}}} = \frac{1}{1} \Rightarrow M = L$$

$$\frac{MP_L}{MP_K} = \frac{w}{r} \Rightarrow \frac{\frac{1}{3}K^{\frac{1}{3}}L^{-\frac{2}{3}}M^{\frac{1}{3}}}{\frac{1}{3}K^{-\frac{2}{3}}L^{\frac{1}{3}}M^{\frac{1}{3}}} = \frac{1}{1} \Rightarrow K = L$$

$$Q = K^{\frac{1}{3}}L^{\frac{1}{3}}M^{\frac{1}{3}}$$

This is a system of three equations in three unknowns. The solution to this system gives us the long-run cost-minimizing input combination:

$$L = Q$$
$$M = Q$$
$$K = Q$$

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b) The tangency condition $\frac{MP_L}{MP_M} = \frac{w}{m}$ is

$$\frac{\frac{1}{3}\overline{K}^{\frac{1}{3}}L^{-\frac{2}{3}}M^{\frac{1}{3}}}{\frac{1}{3}\overline{K}^{\frac{1}{3}}L^{\frac{1}{3}}M^{-\frac{2}{3}}} = \frac{1}{1},$$

which implies

$$M = L$$

To find the short-run cost-minimizing quantity of labor, we plug this back into the production function and solve for L in terms of Q and $\overline{K}$.

$$Q = \overline{K}^{\frac{1}{3}}L^{\frac{1}{3}}L^{\frac{1}{3}}$$

which when we solve for L gives us the short-run cost-minimizing quantity of labor

$$L = \frac{Q^{\frac{3}{2}}}{\overline{K}^{\frac{1}{2}}}$$

Since $M = L$, the short-run cost-minimizing quantity of materials is

$$M = \frac{Q^{\frac{3}{2}}}{\overline{K}^{\frac{1}{2}}}$$

c) Plugging $Q = 4$ into the expressions for the long-run cost-minimizing quantities of labor and materials gives us

$$L = 4$$
$$M = 4$$

Plugging $Q = 4$ and $\overline{K} = 4$ into the expressions for the short-run cost-minimizing quantities of labor and materials gives us

$$L = \frac{4^{\frac{3}{2}}}{4^{\frac{1}{2}}} = 4^{(\frac{3}{2}-\frac{1}{2})} = 4$$
$$M = \frac{4^{\frac{3}{2}}}{4^{\frac{1}{2}}} = 4^{(\frac{3}{2}-\frac{1}{2})} = 4$$

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7.27 a) With three inputs, we need two tangency conditions to ensure that the marginal product per dollar spent is equal across all inputs. (We could write down a third tangency condition, but it would be redundant.) Equating the "bang for the buck" between labor and capital implies $1/2\sqrt{L} = 1/2\sqrt{K}$ or $L = K$. Similarly, equating the "bang for the buck" between labor and materials implies $1/2\sqrt{L} = 1/2\sqrt{M}$ or $L = M$. Then using the production constraint to find the input demand for labor yields $Q = \sqrt{L} + \sqrt{L} + \sqrt{L}$ or $L = (1/3)Q^2$. Since $L = M = K$ from the tangency conditions, we also have $K = (1/3)Q^2$ and $M = (1/3)Q^2$.

b) First, note that with $K = 4$, the firm can produce up to $Q = \sqrt{0} + \sqrt{4} + \sqrt{0} = 2$ units of output without hiring *any* labor or materials. To produce more than $Q = 2$, the firm still balances the marginal product per dollar spent on labor and materials; in part (a), we saw this implied $L = M$. Substituting this and $K = 4$ into the production constraint, we have $Q = \sqrt{L} + \sqrt{4} + \sqrt{L}$ which yields $L = (1/4)(Q - 2)^2$ as the input demand for labor. Then $L = M$ implies that the input demand for materials is $M = (1/4)(Q - 2)^2$. Therefore, the input demand functions are

$$L(Q) = M(Q) = \begin{cases} 0 & Q \le 2 \\ \frac{1}{4}(Q-2)^2 & Q > 2 \end{cases}$$

c) Again, with $K = 4$ and $L = 9$, the firm can produce up to $Q = 5$ units of output without hiring any materials. Should it desire to produce greater levels of output, it can hire materials according to $Q = \sqrt{9} + \sqrt{4} + \sqrt{M}$, or $M = (Q - 5)^2$. Therefore, the input demand for materials is

$$M(Q) = \begin{cases} 0 & Q \le 5 \\ (Q-5)^2 & Q > 5 \end{cases}$$

7.28 The information in the problem tells us that $MP_L = 200$ and $MP_K = 150$ while $w = 25$ and $r = 10$. So $MP_L/w = 8 < MP_K/r = 15$. Thus Acme could maintain its current level of output while reducing costs by employing more capital and less labor. So it is not employing the optimal input bundle.

7.29 We have $MP_L/w = 4/1 = 4 > MP_K/r = 2/2 = 1$. Thus the firm cannot be minimizing its long-run total cost. By employing more labor and less capital, it could maintain 32 units of output while lowering total costs.

7.30 a) Computers are four times as productive as draftsmen; an alternative way of saying this is that $MP_C = 4MP_D$. Since C and D are perfect substitutes, we know the production function has the form $B = aC + bD$, where a and b are positive constants. Thus we can write the production function as $B = C + (1/4)D$. Note that this is consistent with generating one blueprint ($B = 1$) from the following combinations of inputs: $(C, D) = (1, 0)$, $(C, D) = (0, 4)$, and $(C, D) = (0.5, 2)$.

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b) Notice that $MP_C / p_C = 1/10 > MP_D / p_D = 0.25/5 = 1/20$. That is, the marginal product per dollar spent on computer time is always higher than the marginal product per dollar spent on draftsman time. So the optimal input combination involves $D = 0$ and $C = 15$. The graph below illustrates the (dotted) isocost lines with slope = $-p_C / p_D = -2$, along with the (solid) $B = 15$ isoquant with slope = $-MP_C / MP_D = -4$.