Chapter 5

The Theory of Demand

Solutions to Problems

5.3 a) $\varepsilon_{Q,I} = \dfrac{\%\Delta Q}{\%\Delta I} = \dfrac{\Delta Q / Q}{\Delta I / I} = \left(\dfrac{\Delta Q}{\Delta I}\right)\left(\dfrac{I}{Q}\right)$

I and Q must be greater than zero. In addition, assume income increases, *i.e.*, $\Delta I > 0$. If the good is inferior, then $\Delta Q < 0$. Thus, the first term $(\Delta Q / \Delta I) < 0$ and the second term $(I / Q) > 0$. Multiplying these two terms together implies $\varepsilon_{Q,I} < 0$. Inferior goods have a negative income elasticity of demand.

b) If income elasticity of demand is negative then

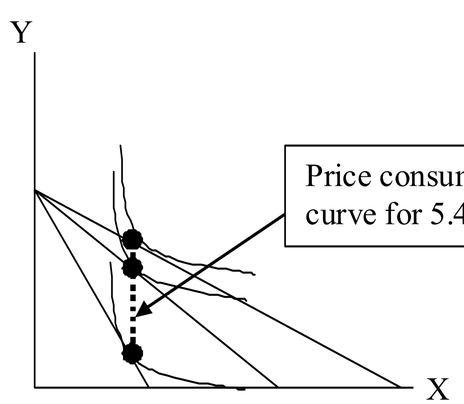
$$\varepsilon_{Q,I} = \left(\frac{\Delta Q}{\Delta I}\right)\left(\frac{I}{Q}\right) < 0 .$$

Since I and Q must be greater than zero, for $\varepsilon_{Q,I}$ to be negative, we must have

$$\frac{\Delta Q}{\Delta I} < 0 .$$

This can only happen if either a) $\Delta Q < 0$ and $\Delta I > 0$ or b) $\Delta Q > 0$ and $\Delta I < 0$. In both instances, the change in quantity demanded moves in the opposite direction as the change in income implying the good is inferior.

5.4



5.5
b) Yes, clothing is a normal good

c) The cross-price elasticity of demand of food with respect to the price of clothing must be zero.

5.6 a) $B = \dfrac{I}{(1.5P_H + P_B)}$

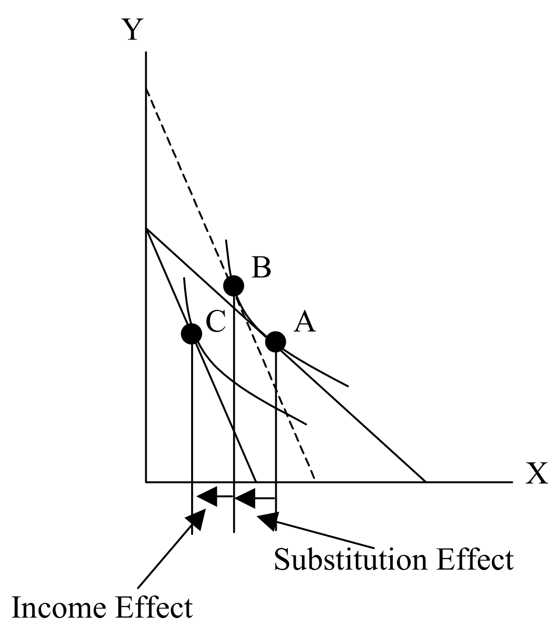
b) The price of hamburgers affects the demand for beer more than the price of beer.

5.7 a) $x = \dfrac{p_y^2}{4p_x^2}$. .

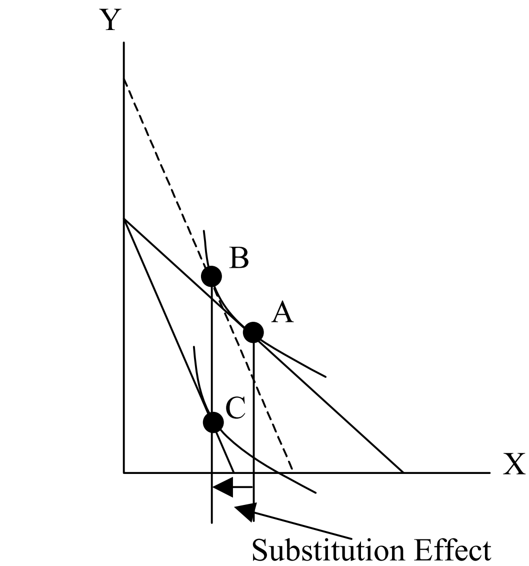
b) $y = \dfrac{I - p_x x}{p_y} = \dfrac{I}{p_y} - \dfrac{p_y}{4p_x}$.

y is a normal good. Moreover, when the price of *x* goes up, the demand for *y* increases as well.

5.8 a)

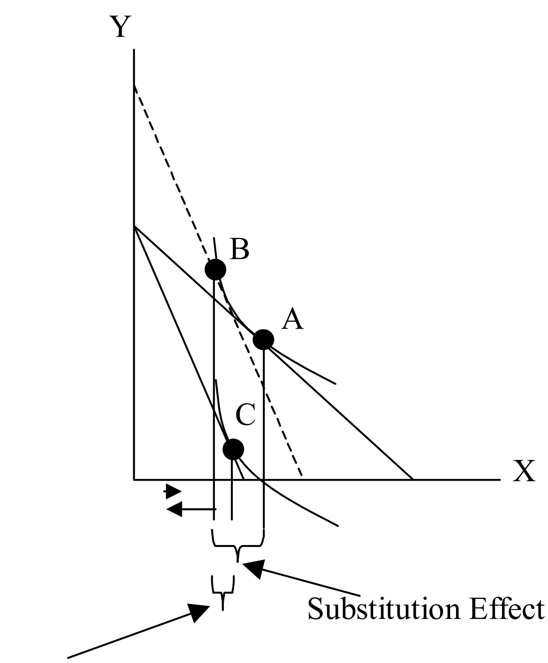


b)

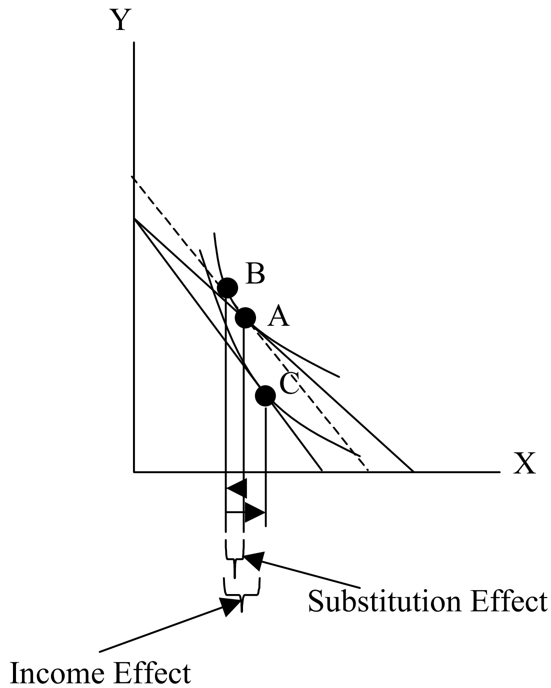


Income Effect = 0

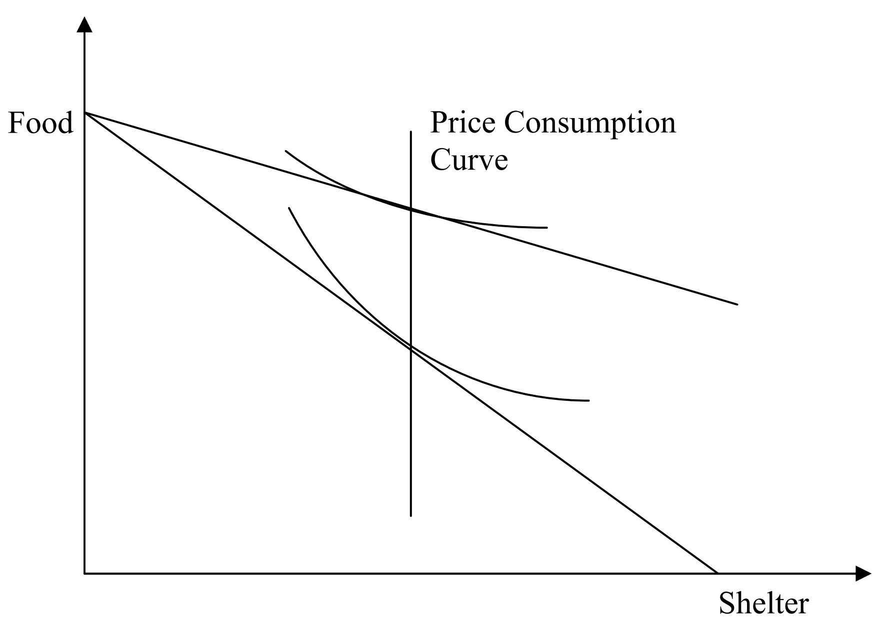
c)



d)



5.9



Reggie's income and substitution effects as a result of a change in the price of shelter must cancel each other out so as to leave a net zero effect.

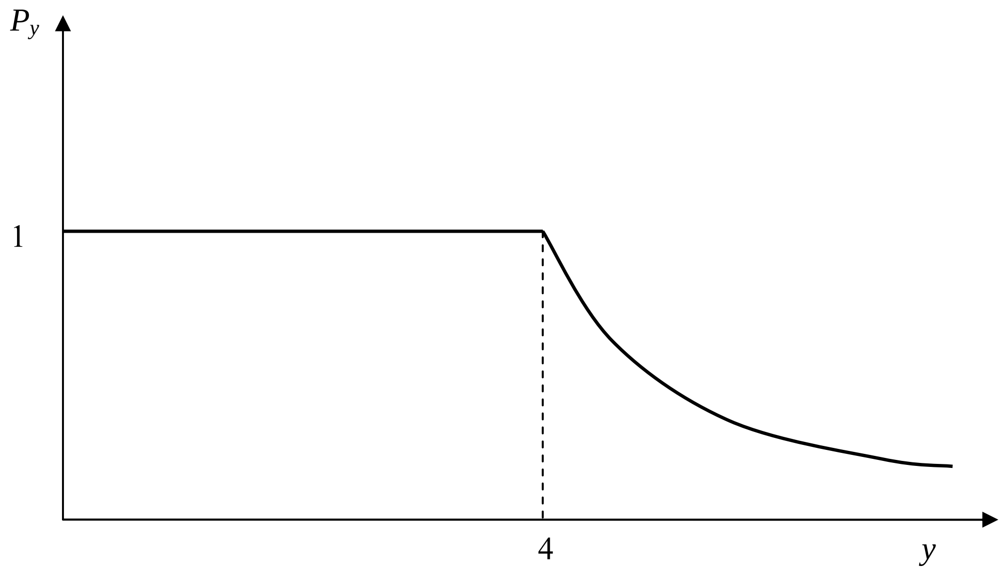
5.10 a) The budget constraint is $8x + 2y = 240$ and the tangency condition is $\frac{2y}{x} = \frac{8}{2} = 4$. Solving, the optimal bundle is (x, y)=(20, 40) with a utility of $20^2(40)$=16,000.

b) Price of x will decrease to 4

5.11 a) All pairs of x and y such that $x + y = 4$ are optimal baskets.

b) nothing but x!!!

c) When price of y is lower than 1 there are zero units of x in the optimal basket. Hence, for $P_x = 1$ and $P_y < 1$ the demand for y equals to I / P_y.



d) By the same argument Ann purchases only x when $P_x = 1$ and $P_y = 1$. Marginal utility per dollar from consumption of x is higher than marginal utility per dollar of y. When $P_x = 1$ and $P_y = 2$ marginal utilities per dollar are the same for both goods. Hence, all baskets such that $2x + y = 4$ are optimal. Construction of the demand curve is similar as in part c).

5.12 Consider any change in income ΔI. For the budget constraint to hold, it must be true that

$$\Delta I = P_x \Delta x + P_y \Delta y .$$

(For example, if income increases then some of it may be spent on x and some on y, but the total new expenditures must be equal to the change in income.) Since we are interested in income elasticities, it helps to rewrite the previous equation as

$$1 = P_x \frac{\Delta x}{\Delta I} + P_y \frac{\Delta y}{\Delta I}$$

Since $\varepsilon_{x,I} = (\Delta x/\Delta I)(I/x)$ and $\varepsilon_{y,I} = (\Delta y/\Delta I)(I/y)$, we can write this as

$$1 = P_x \frac{x}{I}\varepsilon_{x,I} + P_y \frac{y}{I}\varepsilon_{y,I}$$

Or

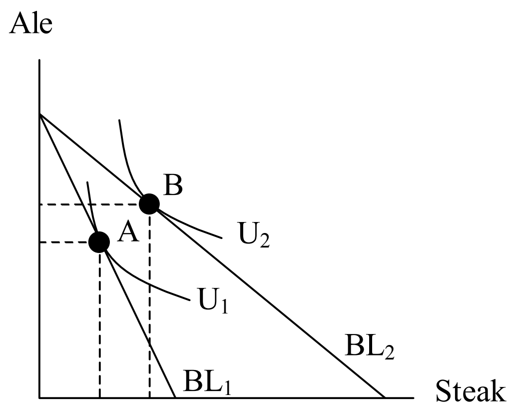
$$I = (P_x x)\varepsilon_{x,I} + (P_y y)\varepsilon_{y,I}$$

But if both goods are luxury goods, then $\varepsilon_{x,I} > 1$ and $\varepsilon_{y,I} > 1$ so that the previous equation implies

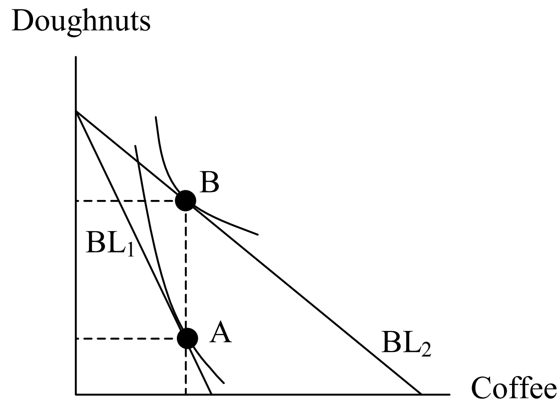
$$I > (P_x x)(1) + (P_y y)(1)$$

Thus, if both *x* and *y* are luxury goods then *I* > *I*, which obviously is untrue! Therefore, both goods cannot simultaneously be luxury goods.

5.13



5.14 a)



.

b) No.

5.15

b) The consumer would only purchase x for prices less than 10.

c)

The indifference curve has a flatter slope than the budget line

d) The consumer would reduce total utility by increasing x above zero.

e)

The location of the demand curve does not depend on P_y .

5.16 As the figure shows, a decrease in the price from p_1 to p_2 induces an increase in quantity from q_1 to q_2. The resulting change in consumer surplus is due to two things:

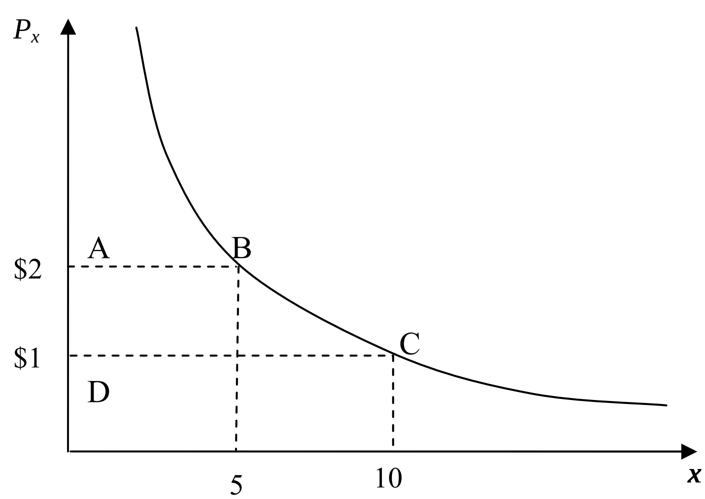
First, the consumer is paying a lower price, per unit, on all the units of the good that he was consuming before the price change. That is, for the q_1 units he was earlier consuming, he now pays a lower price and therefore enjoys a higher consumer surplus, denoted by the area of the rectangle ABCD. Another way of putting this is that if he continued to consume q_1 even after the price change his consumer surplus would increase by only area ABCD.

Second, the lower price induces him to consume more of the good in question. In fact he consumes $(q_2 - q_1)$ *more* units. The additional benefit he gets from this is the area of triangle BDE.

5.17 24

5.18 a) Demand schedule for x is $D(P_x) = 10 / P_x$.

b)



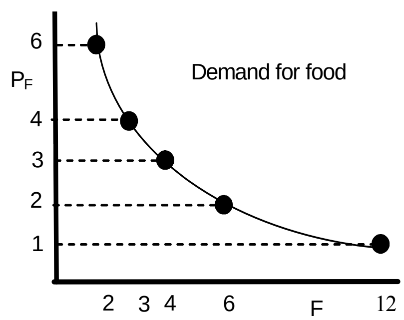
The change in consumer surplus is area of region ABCD under the demand curve. The are of this region can be computed by simple integration: $-\int_{[1,2]} 10/p \, dp = -10 \ln(2)$.

5.19 a) The basket $(x, y) = (15, 60)$.

b) The substitution effect is therefore 17.3 – 15 = 2.3, and the income effect is 20 – 17.3 = 2.7.

5.20 a) $MU_F = C + 1$ $MU_C = F$
Tangency: $MU_F/MU_C = P_F / P_C$. $(C + 1)/ F = P_F/4$ => $4C + 4 = P_FF$. (Eq 1)
Budget Line: $P_FF + P_CC = I$. $P_FF + 4C = 20$. (Eq 2)
Substituting (Eq 1) into (Eq 2): $4C + 4 + 4C = 20$. Thus C = 2, independent of P_F.

From the budget line, we see that $P_FF + 4(2) = 20$, so **the demand for F is F = 12/P_F .**



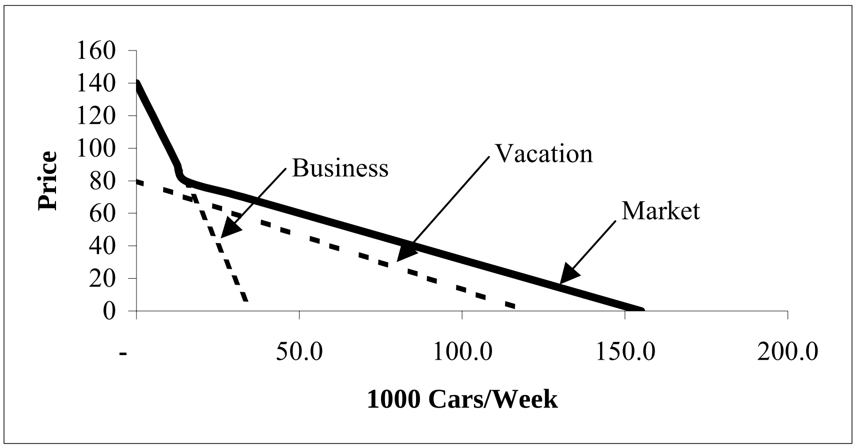
b) Initial Basket: From the demand for food in (a), F = 12/1 = 12, and C = 2.
Also, the initial level of utility is U = FC + F = 12(2) + 12 = 36.
Final Basket: From the demand for food in (a), we know that F = 12/4 = 3, and C = 2. (Also, U = 3(2) + 3 = 9.)
Decomposition Basket: Must be on initial indifference curve, with U = FC + F = 36 (Eq 5)
Tangency condition satisfied with final price: $MU_F/MU_C = P_F / P_C$. $(C + 1)/ F = 4/4$ => $C + 1 = F$. (Eq 3)
Eq 5 can be written as $F(C + 1) = 36$. Using Eq 3, $(C + 1)^2 = 36$, and thus, C = 5.
Also, by Eq 3, F = 6.
So the decomposition basket is F = 6, C = 5.

Income effect on F: $F_{final\ basket} – F_{decomposition\ basket} = 3 – 6 = -3$.
Substitution effect on F: $F_{decomposition\ basket} – F_{initial\ basket} = 6 – 12 = -6$.

5.21 a)

Price ($/day)	Business (000 cars/Week)	Vacation (000 cars/Week)	Market Demand (000 cars/Week)
100	10.0	-	10.0
90	12.5	-	12.5
80	15.0	-	15.0
70	17.5	15.0	32.5
60	20.0	30.0	50.0
50	22.5	45.0	67.5

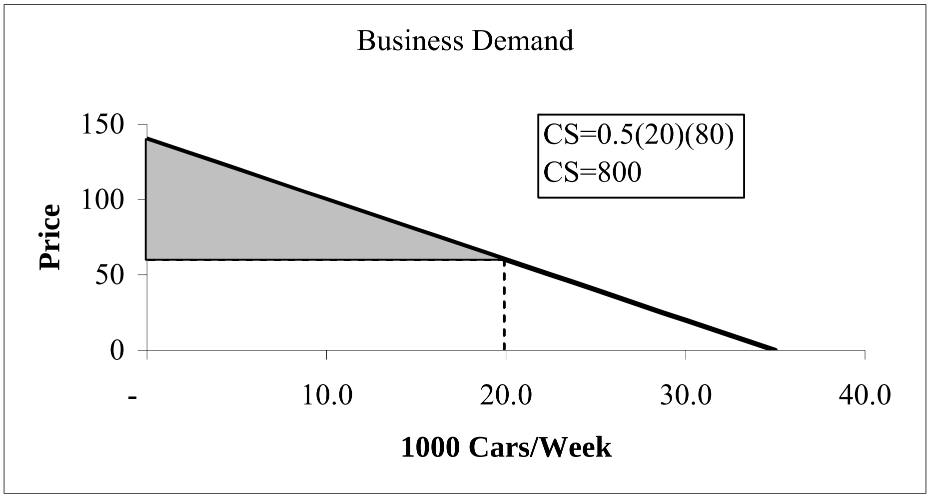
b)

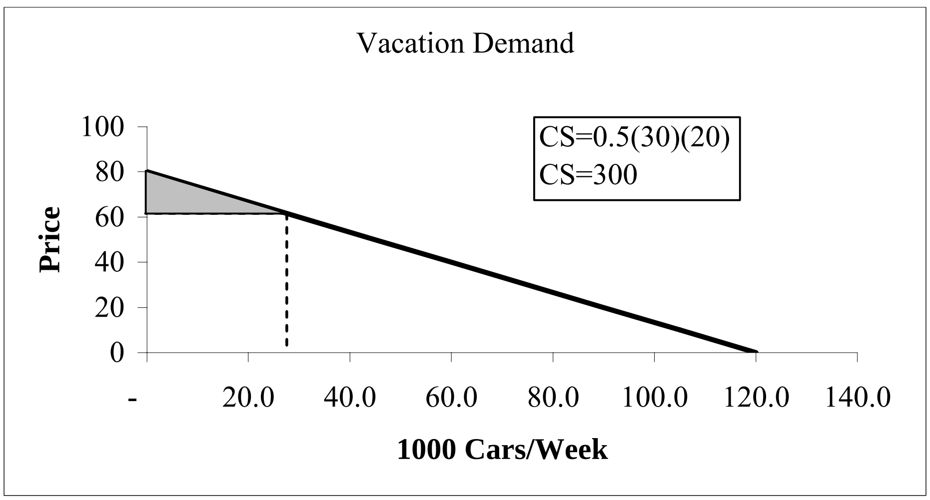


c)

$$Q_m = \begin{cases} 0, & \text{when } P \geq 140 \\ 35 - 0.25P, & \text{when } 80 \leq P < 140 \\ 155 - 1.75P, & \text{when } P < 80 \end{cases}$$

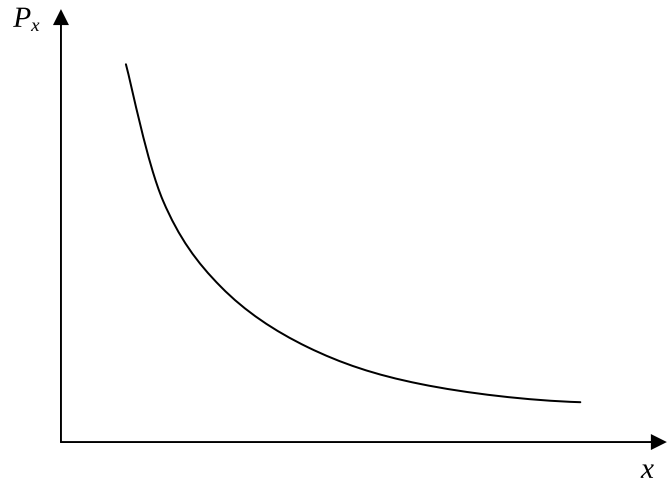
d)





5.22 a) Jim's optimal basket is a solution to equations
$MU_x / MU_y = P / P_y$ and $P x + P_y y = I_J$.
Hence, we have $2xy / x^2 = P$ and $P x + y = 100$
with solution $x = 200 / (3P)$ and $y = 100 / 3$.
Analogous system of equations for Donna is
$y / x = P$ and $P x + y = 150$ with solution $x = 75 / P$ and $y = 75$.

b) Approximate shape of the demand curve for Jim and Donna is depicted below.



c) Aggregate demand is

$$D_x(P) = 200 / (3P) + 75 / P = 445 / (3P).$$

d) aggregate demand is

$$D_x(P) = 200 / (3P) + 75 / P + 75 / P = 650 / (3P).$$

Shape of the demand curve in this case is the same as in part b).

5.23 The market demand and individual demand will have the same price elasticity given any particular price. Denote an individual's demand curve by $Q_i(P)$. With 1,000,000 identical individuals the market demand curve will be $Q_m(P) = 1{,}000{,}000Q_i(P)$. At a given price P, an individual's demand curve will have elasticity $\varepsilon_{Q_i,P} = (\Delta Q_i / \Delta P)(P / Q_i)$. Since $Q_m(P) = 1{,}000{,}000Q_i(P)$, it must also be true that

$$\frac{\Delta Q_m}{\Delta P} = 1{,}000{,}000\frac{\Delta Q_i}{\Delta P}$$

The elasticity for the market demand curve will be

$$\varepsilon_{Q_m,P} = \frac{\Delta Q_m}{\Delta P}\frac{P}{Q_m} = 1{,}000{,}000\frac{\Delta Q_i}{\Delta P}\frac{P}{1{,}000{,}000Q_i} = \frac{\Delta Q_i}{\Delta P}\frac{P}{Q_i} = \varepsilon_{Q_i,P}$$

In other words, with identical consumers the elasticity of the market demand curve will equal the elasticity of the individual demand curve at any price P.

5.24 Bart will only consume when the price is less than 10. Therefore his demand curve for 7-UP is $Q_B = \frac{10-P}{4}$, when P<10 and zero otherwise. Homer will only consume if the price is less than 25 so his demand curve is $Q_H = \frac{25-P}{2}$, when $P < 25$ and zero otherwise.

Therefore the market demand curve for 7-UP as a function of all possible values of price is:

$$Q^M = 0, \text{if } P > 25$$
$$Q^M = \frac{25-P}{2}, \text{if } 10 < P < 25$$
$$Q^M = \frac{60-3P}{4}, \text{if } P < 10$$

5.25 a) If the income consumption curve is vertical the utility function has no income effect. This will occur, for example, with a quasi-linear utility function. This utility function will have the same marginal rate of substitution for any particular value of tea regardless of the level of total utility. If the price of tea falls, flattening the budget line, the consumer will reach a new optimum where the marginal rate of substitution is equal to the slope of the new budget line. Since the budget line has flattened, this cannot occur at the previous optimum amount of tea. The substitution effect implies that this new optimum level of tea will be greater than the previous level. Thus, when the price of tea falls, the quantity of tea demanded increases, implying a downward sloping demand curve. This can be seen in the following figure.