Chapter 3

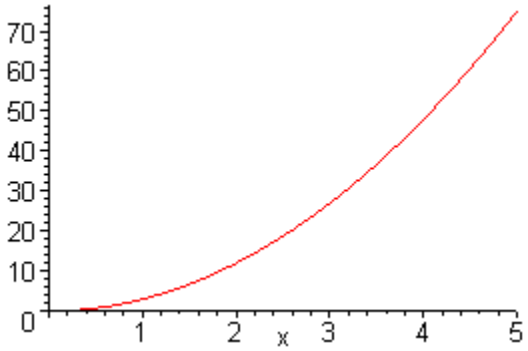
*Preferences and Utility**Solutions to Problems***

3.1 By plugging in ever higher numerical values of x and ever higher numerical values of y, it can be verified that U increases whenever x or y increases.

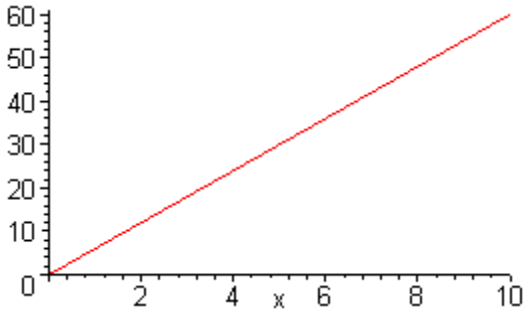
3.2 The two graphs are shown below. It can be seen from both graphs that this function does not satisfy the law of diminishing marginal utility. The first figure shows that utility increases with x, and moreover, that it increases at an increasing rate. For example, an increase in x from 2 to 3, increases utility from 12 to 27 (an increase of 15), while an increase in x from 3 to 4 induces an increase in utility from 27 to 48 (an increase of 21).

This fact is easier to see in the second figure. The marginal utility is an increasing function of x. Higher values of x imply a greater marginal utility. Therefore this function exhibits *increasing* marginal utility.

$$U(x) = 3x^2$$



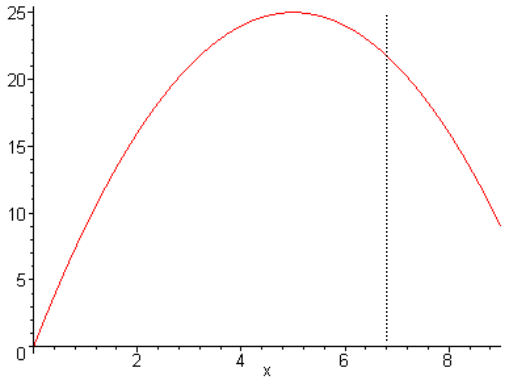
$$MU_x = 6x$$



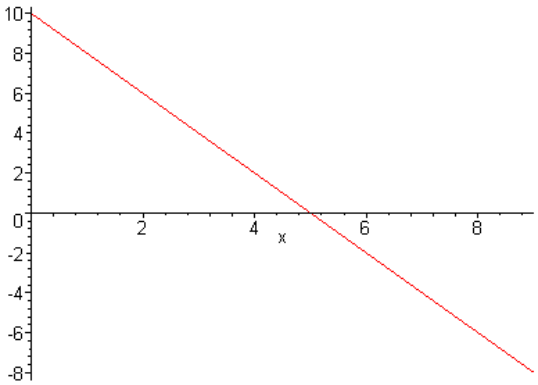
3.3 The first figure below shows Jimmy's utility function for hotdogs. You can see that the point at which $H = 5$ corresponds to the flat portion of the utility function, i.e. the point at which the marginal utility of hotdogs is zero, and beyond which the marginal utility is negative. Alternatively using the second graph it is clear that the point $H = 5$ is when the marginal utility intersects the x-axis, and beyond which it is negative. Both graphs tell you that to maximize his utility Jimmy should only consume 5 hotdogs and not more.

To answer this question algebraically, you should first recognize from the marginal utility function that Jimmy has a diminishing marginal utility of hotdogs. Therefore the point at which he should stop consuming hotdogs is the point at which $MU_H = 0,$ or $10 - 2H = 0.$ This gives $H = 5$.

$U(H) = 10H - H^2$



$MU_H = 10 - 2H$



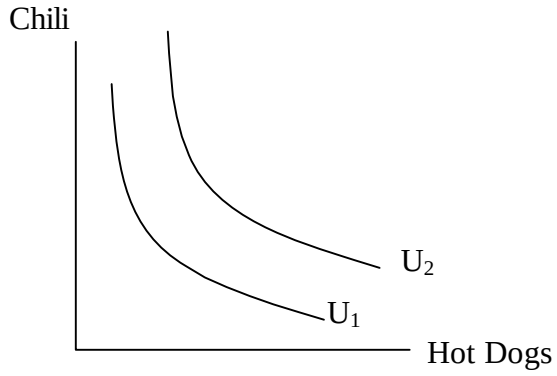
3.4 a) Since U increases whenever x or y increases, more of each good is better. This is also confirmed by noting that MU_x and MU_y are both positive for any positive values of x and y.

b) Since $MU_x = y/2\sqrt{x}$, as x increases (holding y constant), MU_x falls. Therefore the marginal utility of x is diminishing. However, $MU_y = \sqrt{x}$. As y increases, MU_y does not change. Therefore the preferences exhibit a constant, not diminishing, marginal utility of y.

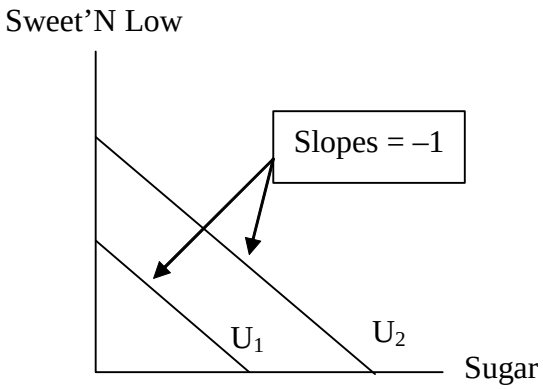
3.5 a) By plugging in ever higher numerical values of x and ever higher numerical values of y, it can be verified that Carlos' utility goes up whenever x or y increases.

b) First consider the marginal utility of x, MU_x. Since x does not appear anywhere in the formula for MU_x, MU_x is independent of x. Hence, the marginal utility of movies is independent of the number of movies seen, and so the marginal utility of movies does not decrease as the number of movies increases. Next consider the marginal utility of y, MU_y. Notice that MU_y is an increasing function of y. Hence, the marginal utility of operas does not decrease in the number of operas seen. In this case, neither good, movies or operas, exhibits diminishing marginal utility.

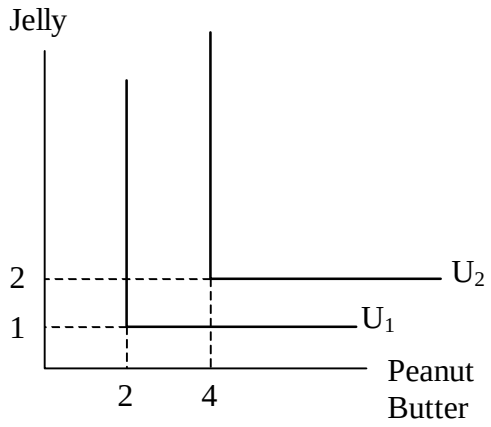
3.6 a)



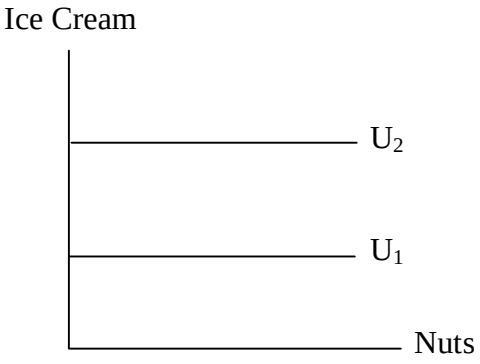
b)



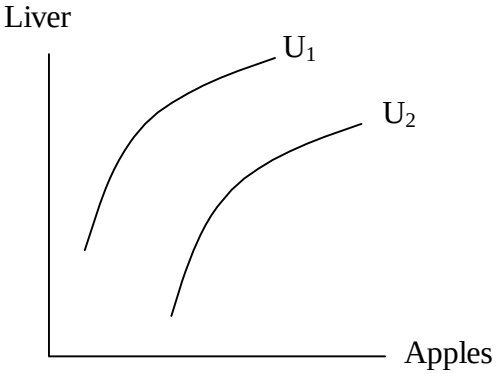
c)



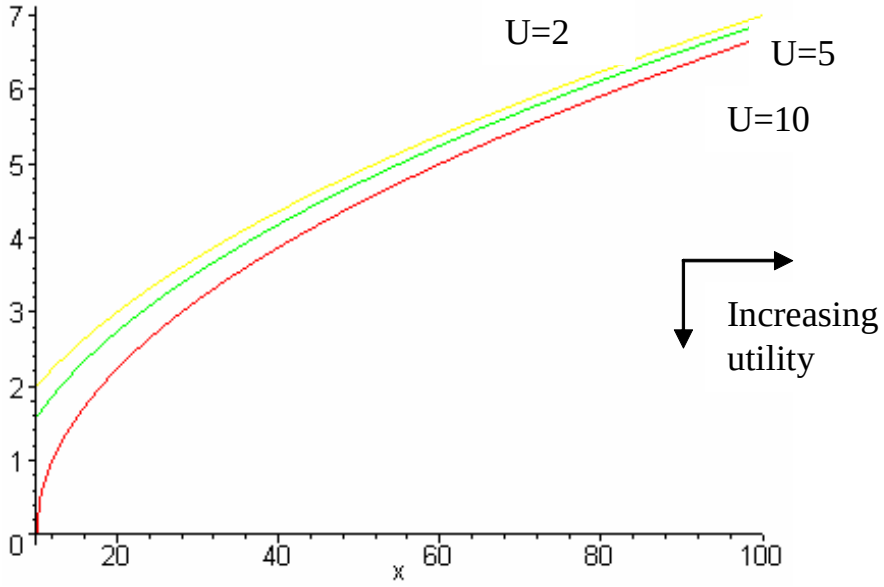
d)



e)



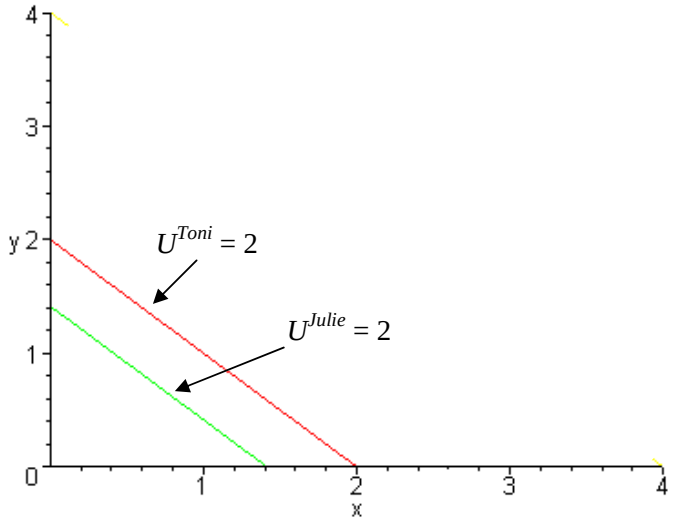
3.7 a) Three indifference curves corresponding to $U = 2$, 5 and 10 are shown in the figure. The direction of increasing utility is down and to the right.



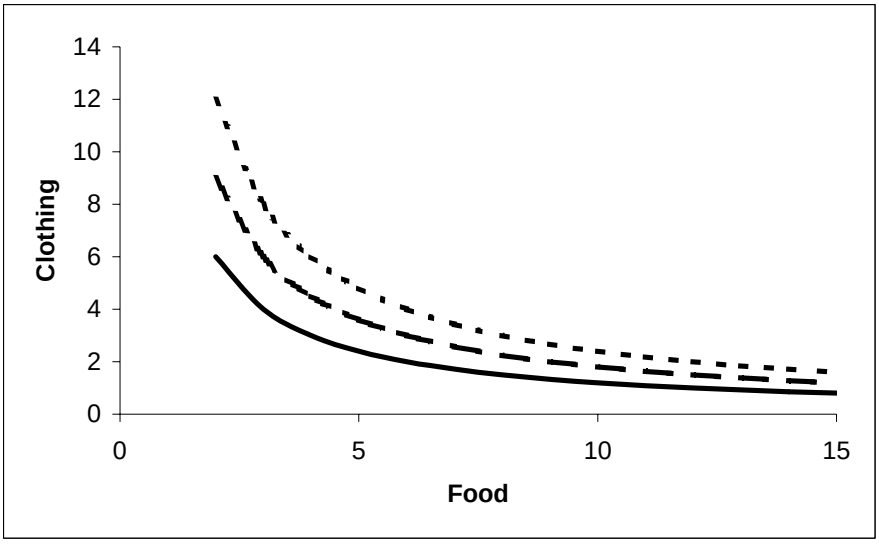
b) Note the negative sign for MU_y. This means that an increase in the consumption of y would *decrease* the consumer's utility. This violates the basic assumption that more is better for this utility function.

3.8 This utility function does have the property of diminishing $MRS_{x,y}$. One way to verify this is to graph several indifference curves. Another way to tell is to use algebra. Recall that $MRS_{x,y} = \frac{MU_x}{MU_y}$. Applying that general formula to this case gives us $MRS_{x,y} = 2\sqrt{y}$. As we move "down" the indifference curve, x increases and y decreases. As y decreases, $2\sqrt{y}$ decreases. Thus, $MRS_{x,y}$ decreases.

3.9 Indifference curves corresponding to $U = 2$ are shown for both Julie and Toni in the graph below. Notice that the indifference curves are parallel everywhere – indeed, $MRS_{x,y} = 1$ for both Julie and Toni, for all values of x and y. Toni's indifference curve for the utility level $U^{Toni} = 2$ is the same as Julie's indifference curve for the utility level $U^{Julie} = 4$. So whenever Julie ranks bundle A higher than bundle B, Toni would have the same ranking, and vice-versa. So Julie and Toni will have the same ordinal ranking of bundles of x and y. (Julie will associate each bundle with a higher utility *level* than Toni will, but that is a *cardinal* ranking.)



3.10 a)

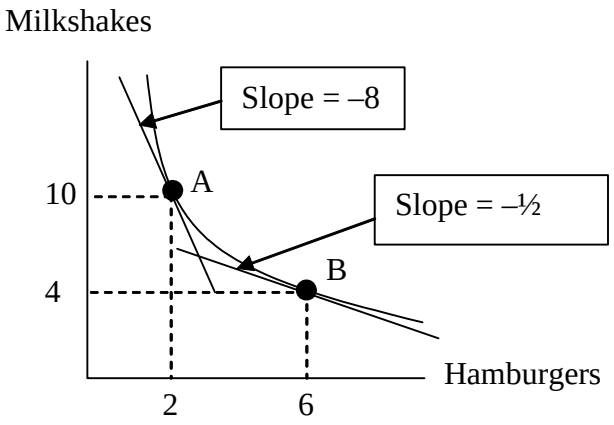


b) Yes, since the indifference curves are bowed in toward the origin we know that $MRS_{F,C}$ declines as F increases and C decreases along an indifference curve.

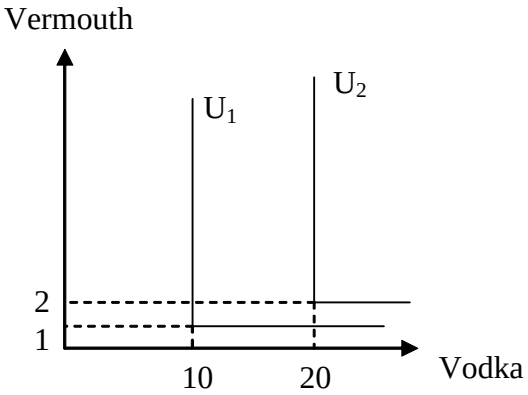
c) $$MRS_{F,C} = \frac{MU_F}{MU_C} = \frac{C}{F}$$

When $F = 2$ and $C = 6$, $MRS_{F,C} = 3$. The slope of the indifference curve is –3. When $F = 4$ and $C = 3$, $MRS_{F,C} = 0.75$, so the slope of the indifference curve is –0.75. Since the marginal rate of substitution falls as F increases and C decreases, she has a diminishing marginal rate of substitution.

3.11

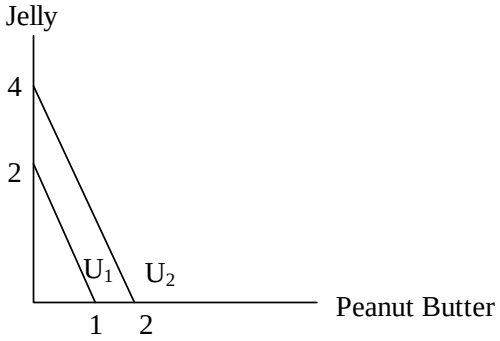


3.12

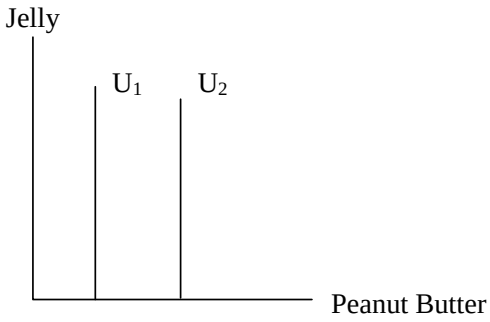


3.13 In the following pictures, $U_2 > U_1$.

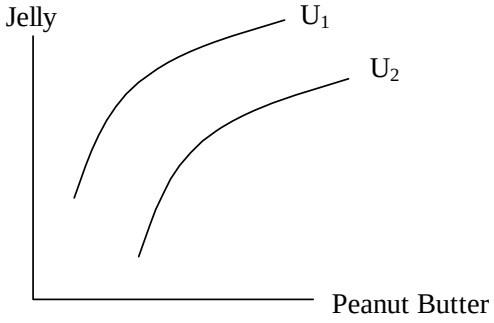
a)



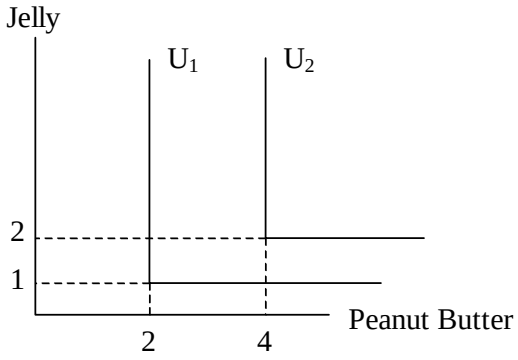
b)



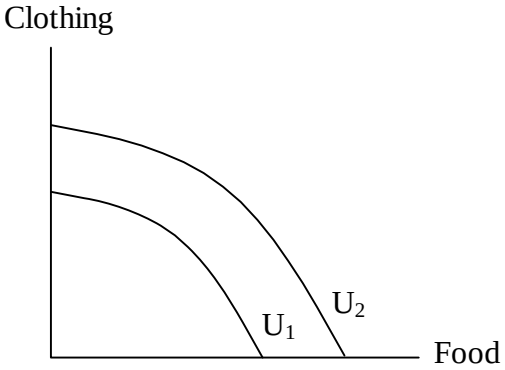
c)



d)



3.14



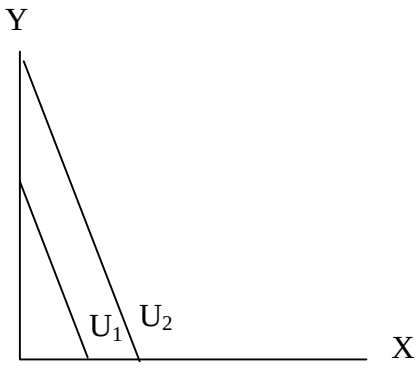
3.15 a) Yes, the "more is better" assumption is satisfied for both goods since both marginal utilities are always positive.

b) The marginal utility of x remains constant at 3 for all values of x.

c) $MRS_{x,y} = 3$

d) The $MRS_{x,y}$ remains constant moving along the indifference curve.

e & f) See figure below



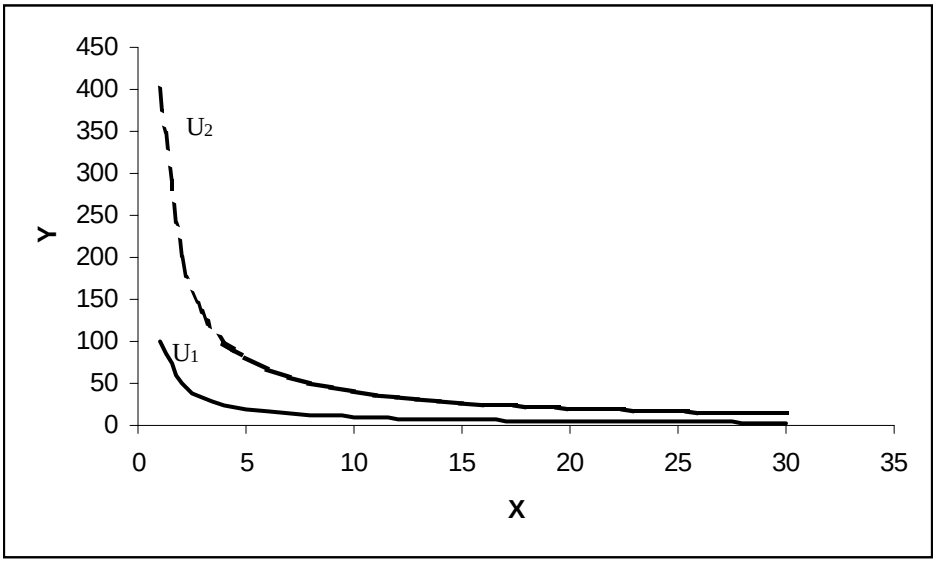
3.16 a) Yes, the "more is better" assumption is satisfied for both goods since both marginal utilities are always positive.

b) The marginal utility of x diminishes as the consumer buys more x.

c) $MRS_{x,y} = \left(\frac{\sqrt{y}}{2\sqrt{x}}\right)\left(\frac{2\sqrt{y}}{\sqrt{x}}\right) = \frac{y}{x}$

d) As the consumer substitutes x for y, the $MRS_{x,y}$ will diminish.

e & f) See figure below. The indifference curves will not intersect either axis.



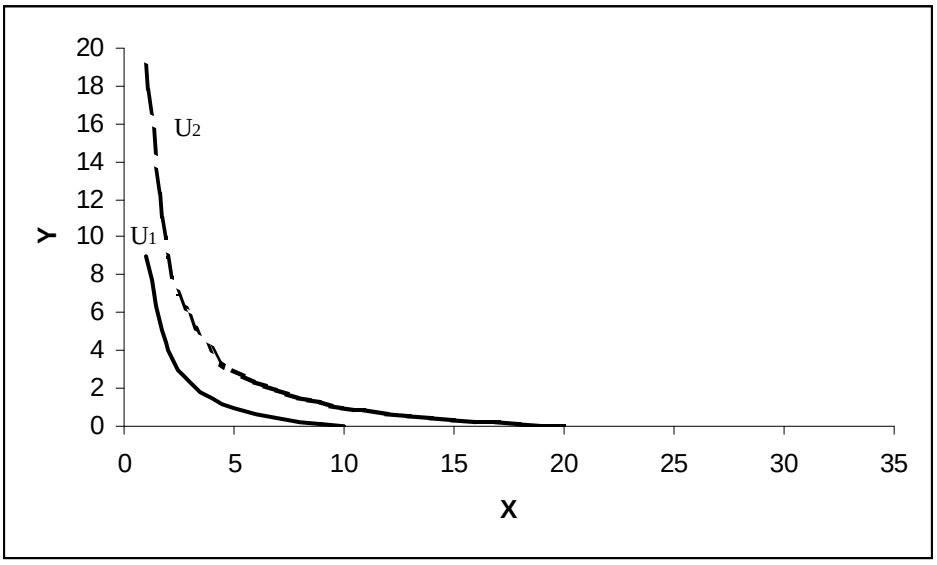
3.17 a) Yes, the "more is better" assumption is satisfied for both goods since both marginal utilities are always positive.

b) The marginal utility of x remains constant as the consumer buys more x.

c) $MRS_{x,y} = \dfrac{y+1}{x}$

d) As the consumer substitutes x for y, the $MRS_{x,y}$ will diminish.

e & f) See figure below. The indifference curves intersect the x-axis, since it is possible that $U > 0$ even if $y = 0$.



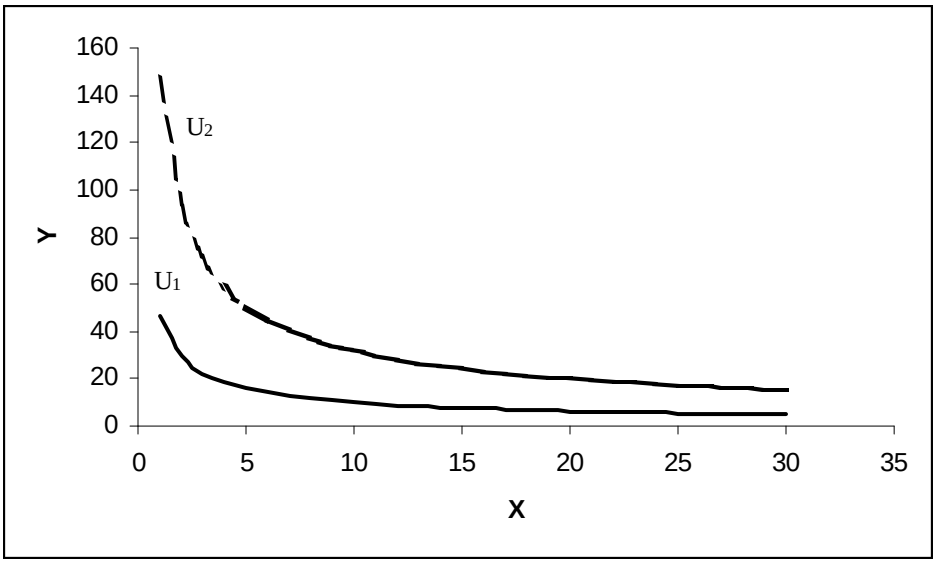
3.18 a) Yes, the "more is better" assumption is satisfied for both goods since both marginal utilities are always positive.

b) The marginal utility of x diminishes as the consumer buys more x.

c) $MRS_{x,y} = \dfrac{.4(y^{0.6}/x^{0.6})}{.6(x^{0.4}/y^{0.4})} = \dfrac{0.4y}{0.6x}$

d) As the consumer substitutes x for y, the $MRS_{x,y}$ will diminish.

e & f) See figure below. The indifference curves do not intersect either axis.



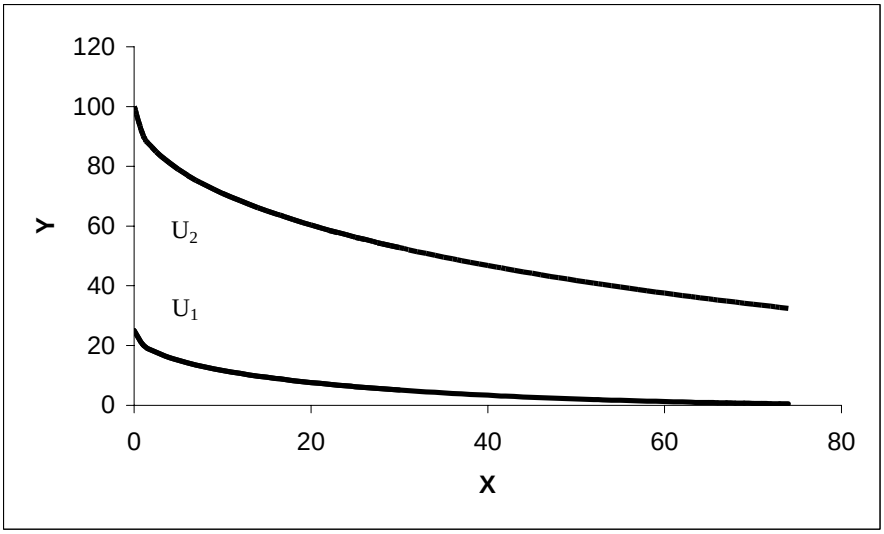
3.19 a) Yes, the "more is better" assumption is satisfied for both goods since both marginal utilities are always positive.

b) The marginal utility of x diminishes as the consumer buys more x.

c) $MRS_{x,y} = \dfrac{1/(2\sqrt{x})}{1/\sqrt{y}} = \dfrac{\sqrt{y}}{2\sqrt{x}}$

d) As the consumer substitutes x for y, the $MRS_{x,y}$ will diminish.

e & f) See figure below. Since it is possible to have $U > 0$ if either $x = 0$ (and $y > 0$) or $y = 0$ (and $x > 0$), the indifference curves intersect both axes.



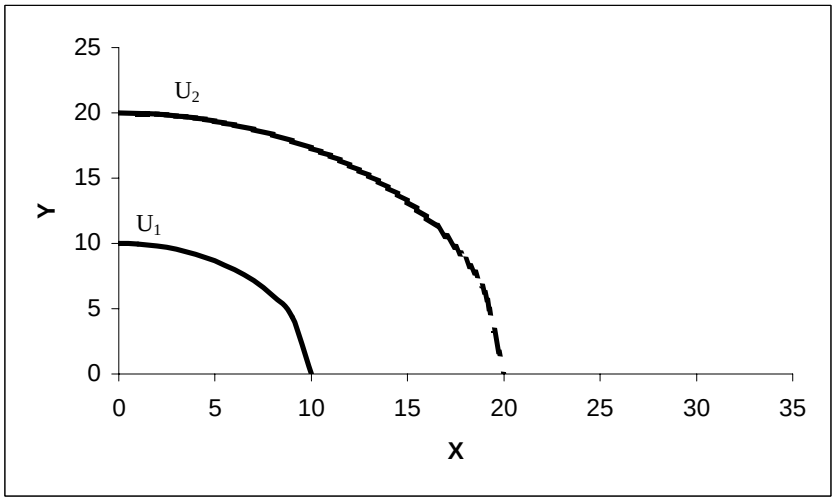
3.20 a) Yes, the "more is better" assumption is satisfied for both goods since both marginal utilities are always positive.

b) The marginal utility of x increases as the consumer buys more x.

c) $MRS_{x,y} = \dfrac{2x}{2y} = \dfrac{x}{y}$

d) As the consumer substitutes x for y, the $MRS_{x,y}$ will increase.

e & f) See figure below. Since it is possible to have $U > 0$ if either $x = 0$ (and $y > 0$) or $y = 0$ (and $x > 0$), the indifference curves intersect both axes.



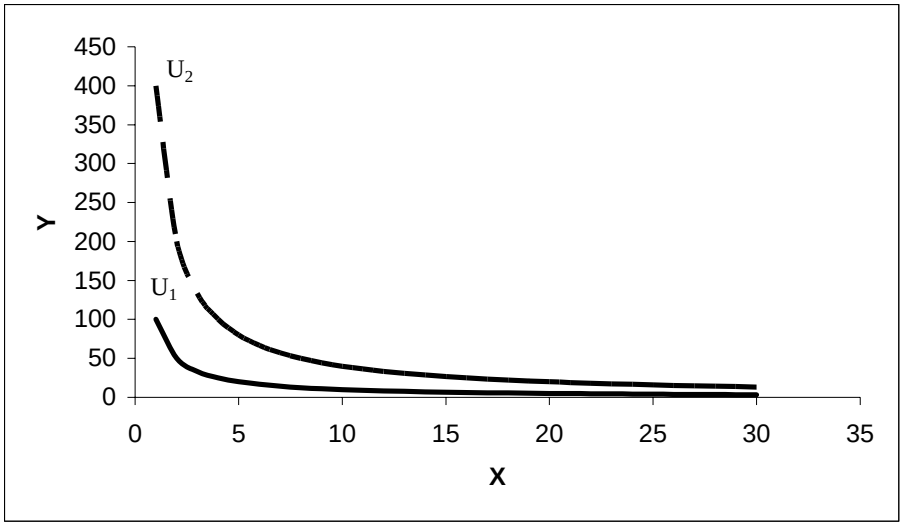
3.21 a) Yes, the "more is better" assumption is satisfied for both goods since both marginal utilities are always positive.

b) Since we do not know the value of α, only that it is positive, we need to specify three possible cases:
When $\alpha < 1$, the marginal utility of x diminishes as x increases.
When $\alpha = 1$, the marginal utility of x remains constant as x increases.
When $\alpha > 1$, the marginal utility of x increases as x increases.

c) $MRS_{x,y} = \dfrac{\alpha A x^{\alpha-1} y^{\beta}}{\beta A x^{\alpha} y^{\beta-1}} = \dfrac{\alpha y}{\beta x}$

d) As the consumer substitutes x for y, the $MRS_{x,y}$ will diminish.

e & f) The graph below depicts indifference curves for the case where $A = 1$ and $\alpha = \beta = 0.5$. Thus $U(x, y) = x^{0.5} y^{0.5}$. Regardless, the indifference curves will never intersect either axis.



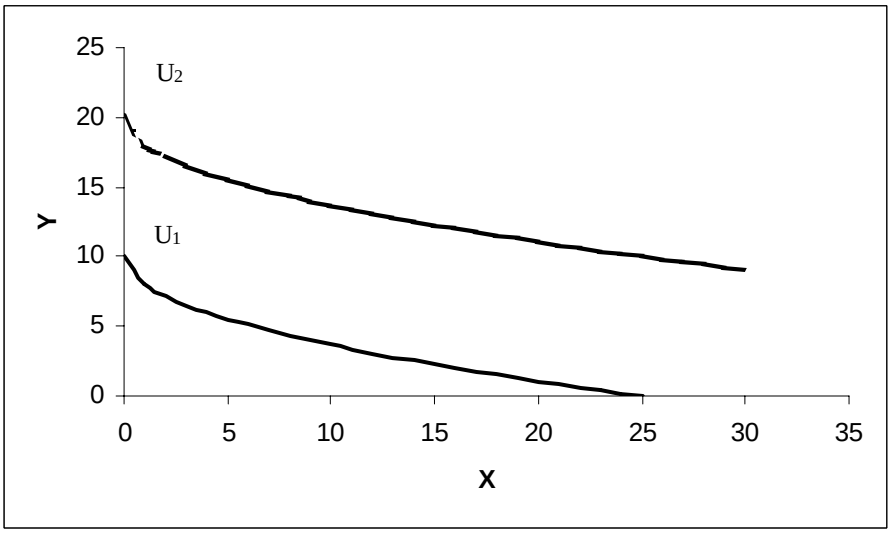
3.22 a) Yes, the "more is better" assumption is satisfied for both goods since both marginal utilities are always positive.

b) The marginal utility of x increases as the consumer buys more x.

c) $MRS_{x,y} = \dfrac{1/\sqrt{x}}{1} = 1/\sqrt{x}$

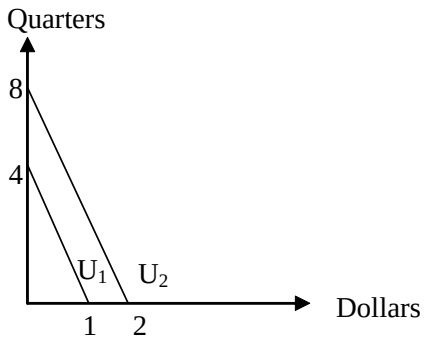
d) As the consumer substitutes x for y, the $MRS_{x,y}$ will diminish.

e) Since it is possible to have $U > 0$ if either $x = 0$ (and $y > 0$) or $y = 0$ (and $x > 0$), the indifference curves intersect both axes.

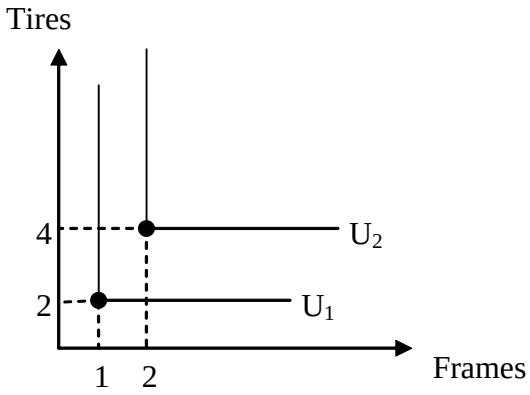


f) The slope of a typical indifference curve at some basket (x, y) is the $MRS_{x,y} = 1/\sqrt{x}$. At $x = 4$, $MRS_{x,y} = 1/\sqrt{4} = 0.5$. Note that this holds regardless of the value of y. Therefore, the slope of any indifference curve at $x = 4$ will be -0.5.

3.23 a) Quarters and dollars are perfect substitutes:



b) Tires and frames are perfect complements:



3.24 First, the expression for $MRS_{x,y}$ is

$$MRS_{x,y} = \frac{MU_x}{MU_y} = \frac{\alpha x^{\alpha-1} y^{1-\alpha}}{(1-\alpha) x^{\alpha} y^{-\alpha}} = \frac{\alpha}{1-\alpha}\frac{y}{x}$$

Since we know that $MRS_{x,y} = 4$ when $x = 4$ and $y = 8$,

$$4 = \frac{\alpha}{1-\alpha}\frac{8}{4}$$

$$2 = \frac{\alpha}{1-\alpha}$$

$$2 - 2\alpha = \alpha$$

$$\alpha = \frac{2}{3}$$

3.25 Recall that $MRS_{x,y} = \frac{MU_x}{MU_y}$. Substituting in the marginal utilities given above yields $MRS_{x,y} = \frac{x^{\rho-1}}{y^{\rho-1}}$. Now, because $\rho < 1$, $x^{\rho-1}$ decreases as x increases. By the same logic, $y^{\rho-1}$ increases as y decreases. As we "slide down" an indifference curve, x increases and y decreases, so it follows that $MRS_{x,y}$ decreases. Thus, this utility function exhibits diminishing marginal rate of substitution of x for y.

