The Optimal Taxation Approach to Intergovernmental Grants

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Abstract

An optimal tax system equates the marginal cost of public funds across all tax bases. This idea is applied to a federation to derive the optimal unconditional transfers that will promote an optimal allocation of taxation and expenditures among the governments in the federation. This approach provides insights into the concepts of vertical and horizontal fiscal imbalance, fiscal capacity, and fiscal need. Expressions for the optimal fiscal equalization grant and the optimal vertical fiscal gap are derived. We also show how the marginal cost of public funds affects the optimal matching grant rate for activities that generate expenditure externalities.

Key words:

fiscal federalism, intergovernmental grants, equalization grants, matching grants, fiscal imbalance, fiscal capacity, fiscal need, marginal cost of public funds.

1.0 Introduction

Most reviews of the literature on intergovernmental grants rarely mention the theory of optimal taxation and expenditures, which was developed in the 1970s and 1980s and which provides the welfare foundations for tax reform and cost-benefit analysis. Aside from a passing reference to Roger Gordon's 1983 paper "An Optimal Tax Approach to Fiscal Federalism," there are no references to the optimal tax literature in Oates (1999, 2005), and none in Shah (2007), Boadway (2007), or Spahn (2007). Evidence of this neglect is contained in Oates' description of the "First Generation" theory of fiscal federalism as based on the work of Samuelson, Musgrave, and Arrow—all towering figures in the 1950s and 1960s public finance literature—and the emerging "Second Generation" theory of fiscal federalism which emphasizes "political processes and the behavior of political agents" and "problems of information". Clearly missing from Oates' description of the building blocks of the theory of fiscal federalism are the key works on optimal taxation and expenditure policies by Diamond and Mirrlees (1971), Stiglitz and Dasgupta (1971), Atkinson and Stern (1974), and Diamond (1976) and the literature on the reform of fiscal policies, largely based on the concept of the marginal cost of public funds, in Wildasin (1984), Browning (1987), Triest (1990), Mayshar (1991), and Ballard and Fullerton (1992) to name but a few of the key references in this large literature.

The failure to incorporate the key ideas and concepts of the optimal taxation literature in the theory of fiscal federalism (except perhaps with regard to tax competition) is puzzling because the optimal tax literature provides a solid welfare foundation for the analysis of many of the key aspects of federalism, especially intergovernmental grants. For the most part, the optimal tax approach to intergovernmental grants complements, rather than conflicts, with the conventional discussion of the role of intergovernmental transfers in a federation. It can shed light on the previously ill-defined concepts of fiscal imbalance, fiscal capacity, and fiscal needs, and it provides an analytical foundation for the provision of unconditional grants to address vertical and horizontal fiscal imbalances in a federation and for the matching rates for conditional grants to address expenditure externalities.

The key concept in the optimal taxation/tax reform literature is the marginal cost of public funds (MCF) which is a measure of the burden that is imposed on a society when the government raises an additional dollar of tax revenue through a tax rate increase. The additional burden includes the marginal welfare loss that arises because a tax rate increase causes a further distortion in the allocation of a society's resources. Equity concerns are normally at least as important as efficiency issues, and the MCF concept can be augmented to reflect the distributional effects of taxes on different income groups in a society and distributional weights used to incorporate these losses in the measured social loss from a tax increase. We will refer to the distributionally-weighted MCF as the social marginal cost of public funds (SMCF). As we will argue below, these concepts can play a key role in developing a normative theory of intergovernmental grants.

In Section 2.1, we describe the welfare foundations for a system of unconditional intergovernmental grants that addresses the problems of vertical and horizontal fiscal imbalance in a federation. *Vertical fiscal balance in taxation* is achieved when the SMCF is equalized between the central government and the subnational governments and h*orizontal fiscal balance in taxation* occurs when the SMCF is the same for all subnational governments. There is a v*ertical fiscal balance in expenditures* when the ratio of the distributionally-weighted marginal benefits of the public services provided by a subnational government and the central government is equal to their relative marginal costs of production. *Horizontal fiscal balance in expenditures* occurs when the ratio of the distributionally-weighted marginal benefits from the public services provided by any two subnational governments is equal to the ratio of their marginal costs of production. Fiscal imbalances occur when the SMCFs are not equalized between levels of government or across subnational governments, and federations usually require systems of unconditional transfers to eliminate these imbalances.

In Section 2.2, we derive an expression for the optimal equalization grants to eliminate horizontal fiscal imbalances by equalizing the SMCFs among the subnational governments. The optimal equalization grant to a subnational government will be decreasing with its *fiscal capacity* and increasing in its *expenditure needs*. Fiscal capacity is the ability of a government to raise revenues at a low MCF. It depends not only on the size of a government's tax base, but also on the tax sensitivity of its tax base. The greater the tax sensitivity of the base, the lower a government's fiscal capacity because its marginal cost of public funds will be higher. The conventional measure of fiscal capacity—the size of a government's tax bases relative to some standard—is at best an approximation to a government's true fiscal capacity and may exaggerate the true fiscal capacities of small subnational governments. Expenditure needs are defined (within the context of the optimal equalization grant formula) as the amount a subnational government would spend on providing public services if it had the same SMCF as other subnational governments. The cost of producing public services, the strength of the preference for local public services, equity considerations, and the SMCF will jointly determine the expenditure needs of a subnational government. The SMCF affects the expenditure needs component of the optimal equalization grant because it reflects "affordability" and therefore determines how generous the equalization program will be. If all subnational governments have the same expenditure needs and their tax bases are equally tax sensitive, the optimal equalization grant formula will be the same as the representative tax system (RTS) equalization grant formula, which can be thought of as an approximation to the optimal equalization grant.

The *vertical fiscal gap* in a federation can be defined as the proportion of subnational government spending that is financed by transfers from the central government. In Section 2.3, we derive an expression for the optimal vertical fiscal gap, which equalizes the MCFs of the two levels of government in a federation. We show that the optimal fiscal gap implements the *Ramsey Rule* for optimal taxation in a federal context—the tax bases of the two levels of government should be reduced in the same proportion. Under certain conditions, the optimal fiscal gap also implements the *Inverse Elasticity Rule* of optimal taxation, which implies that each level of government's tax rate should be inversely proportional to the elasticity of its tax base. Since subnational governments' tax bases are usually more tax sensitive than the central government's tax bases, the optimal fiscal system will involve relatively high taxes at the federal level and transfers to subnational governments. Only rarely will the optimal fiscal gap be zero, i.e. each level of government raises enough revenue to cover its own expenditures. The optimal fiscal gap will be higher when (a) the central government's expenditure responsibilities are lower, (b) the subnational governments' expenditure responsibilities are higher, (c) the central government's fiscal capacity is higher, or (d) when the subnational governments' fiscal capacities are lower. Consequently, a country's expenditure assignment and tax assignment will determine its optimal fiscal gap.

Expenditure externalities occur when the public services provided by a subnational government affect the residents of other subnational governments. Matching grants from the central government can correct the potential biases in subnational governments' expenditure decisions by altering the "prices" that they face. The conventional rule is to set the matching grant rate equal to the proportion of the total marginal benefit of the activity that accrues to non-residents. In Section 3.0, we consider how the *optimal matching grants* need to be modified to take into account the SMCFs of the central and subnational governments. If the federation has achieved vertical and horizontal fiscal balance through a system of unconditional intergovernmental transfers, the optimal matching rate is equal to the distributionally-weighted direct external effect and the indirect external effect on the revenues of other subnational governments, valued at the common SMCF, as a percentage of the total social marginal benefit from the activity. The matching rate will be increasing in the SMCF if the activity reduces the revenues of the subnational government that undertakes the activity. The matching rate will normally be increasing with the SMCF when the activity increases the revenue of other jurisdictions. We also show that if there is a vertical fiscal imbalance in a federation, where the central government has a lower SMCF than the subnational governments, then the matching rate will be increased because there is a social gain from shifting more of the costs of the activity to the central government because it can finance the activity at a lower social cost. On the other hand, if there is horizontal fiscal imbalance, where the activity is undertaken by a "rich" subnational government with a low SMCF, then the matching rate will be reduced

2.0 Fiscal Imbalances and Unconditional Grants

A fiscal imbalance has often been defined as a mismatch between a government's spending responsibilities and its access to tax revenues. See, for example, Breton (1996, p.197). This definition of a vertical fiscal imbalance implies that an entire tier of subnational governments is unable to fund their spending responsibilities from their own revenue sources. Taken literally, this would imply that all governments are at the top of their Laffer curves and cannot raise any more revenues by raising their tax rates. While this might occur in some rare situations, it is surely not a generic problem. Subnational governments can almost always raise more tax revenue by increasing their tax rates, but they do not want to because the marginal cost of raising additional tax revenue is very high. A further problem with the conventional definition is that while the spending responsibilities of the subnational governments may be listed in a country's constitution, the level of spending and the quality of these services are not specified. Consequently, we can never really determine whether governments have met their "spending responsibilities". Confronted with these shaky conceptual foundations, empirical studies of vertical fiscal imbalance have simply measured the extent to which subnational government spending is financed by central government transfers.[1] However, this is an

[1] A recent OECD (2007, p.8) report on intergovernmental transfers defines a vertical fiscal imbalance as "the difference between own tax revenue and own expenditure of a jurisdiction." See Rodden and Wibbels (2002), and Bird and Tarasov (2004) for empirical studies of the vertical fiscal gap.

accounting measure of fiscal imbalance, not an economics-based measure of an imbalance in the allocation of spending and taxation in a federation. In the following section, we provide an analytical foundation for the concept of vertical and horizontal fiscal balance in a federation.

2.1 A Model of Optimal Unconditional Grants

To keep the notation as simple as possible, we assume a federation with two levels of government—a central government and n subnational governments which could be states or municipalities. The fiscal variables of the central government are denoted by a subscript 0 and the subnational governments' fiscal variables by subscripts 1 to n. Each subnational government is assumed to have a homogeneous population that can be represented by the income and preferences of a single resident. The total population of the federation is one unit. The population of jurisdiction i is immobile and equal to f_i. Each government has a tax base, B_i, levies a tax rate, t_i, on this base, and provides a public service, g_i, at a constant unit production cost of c_i, where i = 0, 1, …n. Let T_i be the per capita lump-sum transfer received by ($T_i > 0$), or paid by ($T_i < 0$), subnational government i. The budget constraints of the central government and a representative subnational government are given below:

$$t_0 B_0 - c_0 g_0 - \sum_{i=1}^{n} f_i T_i = 0 \tag{2.1}$$

$$t_i B_i + T_i - c_i g_i = 0 \qquad i = 1, 2, ..., n \tag{2.2}$$

The central government could be a net recipient of the lump-sum transfers if the population-weighted sum of the per capita subnational governments' transfers is negative. Thus we envisage the possibility that one or more subnational governments makes transfers to the central government, although in most federations transfers flow from the central government to the subnational governments.

The well-being of a resident of subnational government i is reflected in the following reduced-form indirect utility function:

$$V^i = V(t_0, t_i, g_0, g_i, Y_i) \tag{2.3}$$

where Y_i is the lump-sum income that a resident of subnational government i may receive. The existence of lump-sum income is mainly a heuristic device that allows us to define a resident's marginal utility of income, $\partial V_i/\partial Y_i = \lambda_i > 0$. Based on duality theory, the partial derivatives of the indirect utility function with respect to the tax rates are:

$$\frac{\partial V^i}{\partial t_0} = -\lambda_i B_{i0} \tag{2.4}$$

$$\frac{\partial V^i}{\partial t_i} = -\lambda_i B_i \tag{2.5}$$

where B_{i0} is the central government's per capita tax base in subnational government i. The central government's total per capita tax base is equal to $B_0 = \sum_{i=1}^{n} f_i B_{i0}$. We will assume that each government's tax base is decreasing in its tax rate, $\partial B_i/\partial t_i < 0$, i = 0, 1, …, n. Our assumption that there are no tax externalities in the federation implies that $\partial B_i/\partial t_j = 0$ for $i \neq j$. In particular, a subnational government's tax rate does not affect the tax bases of the other subnational government or the central government, and the central government's tax rate does not affect the tax bases of the subnational governments.[2] Furthermore, we assume that a tax rate increase in one subnational government does not affect the input or output prices faced by the residents of another jurisdiction. That is, $\partial V_i / \partial t_j = 0 \;\; i \neq j, j = 1, 2, \ldots, n$. Finally, note that we are assuming that the central government has to levy the same tax rate, t_0, across the entire federation.

We will find it convenient to define the marginal benefit that a resident of subnational government i receives from the central government's public service and from its own subnational government's public service as:

$$MB^i_{g_0} = \frac{1}{\lambda_i}\frac{\partial V^i}{\partial g_0} > 0 \tag{2.6}$$

$$MB^i_{g_i} = \frac{1}{\lambda_i}\frac{\partial V^i}{\partial g_i} > 0 \tag{2.7}$$

The absence of interjurisdictional benefit spillover effects is implied by our assumption that $\partial V^i / \partial g_j = 0$ for $i \neq j > 0$. It also assumed that the public services do not affect the governments' tax bases.

The marginal cost of public funds represents the cost to the private sector raising an extra dollar of tax revenue through a tax rate increase and is represented by the following expression:[3]

$$MCF_i = \frac{B_i}{\frac{d(t_i B_i)}{dt_i}} = \frac{B_i}{B_i + t_i \frac{dB_i}{dt_i}} = \frac{1}{1 + t_i \eta_i} \tag{2.8}$$

where $\eta_i = d\,ln\,B_i / dt_i < 0$. The numerator reflects the fact that the harm to households from a small tax rate increase is proportional to the tax base. The denominator is the rate

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[2] These are very strong assumptions. In this section we adopt a very simple framework in order to focus attention on the key issue of the role of intergovernmental grants in resolving the problems of vertical and horizontal fiscal imbalances.

[3] There is a very large literature on the interpretation and measurement of the MCF. See Dahlby (2008) for comprehensive treatment of the concept and measurement of the MCF.

of increase in tax revenues from a small tax rate increase, and therefore the ratio represents the cost to the society of raising an additional dollar of tax revenue through a tax rate increase. We assume that a higher tax rate causes the tax base to shrink because of tax avoidance or tax evasion, and therefore $MCF_i > 1$ for $t_i > 0$. It is also assumed that the government is always on the upward-sloping section of its Laffer curve and therefore $1 + t_i\eta_i > 0$. At the revenue-maximizing tax rate, $t_i = -1/\eta_i$, the MCF_i would be infinite because a small tax rate increase would impose a burden on households without raising any additional revenues. The MCF_i has the simple form given in (2.8) because we have assumed that there are no non-tax distortions in the economy, i.e. no externalities from the production or consumption of private goods and services, no monopoly price distortions, and no market distortions caused by asymmetric information.

The ethical standard that we adopt for evaluating a federation's fiscal structure is the set of fiscal policies that a social planner, maximizing the following social welfare function, would choose:

$$W = W\left(f_1 V^1, f_2 V^2, ..., f_n V^n\right) \tag{2.9}$$

This social welfare function is based on the population-weighted well-being of the representative individuals in each subnational government. The social welfare function implicitly expresses the society's willingness to trade off the well-being of individuals in different jurisdictions. We will define the distributional weights that a society places on an additional dollar received by an individual in subnational government i as $\beta_i = (\partial W/\partial V^i)\lambda_i$. Usually we think that the social welfare function expresses a society's concern for social justice, and this will be reflected in pro-poor distributional weights such that the βs are higher for individuals with lower utility levels. Here, however, the individuals also differ by location, and the social welfare function might express some regional preferences that go beyond income comparisons.

The Langrangian for the social planner's problem is given below:

$$\Lambda = W\left(f_1V^1, f_2V^2, \ldots. f_nV^n\right) + \mu_0\left(t_0B_0 - \sum_{i=1}^{n} f_iT_i - c_0g_0\right) + \sum_{i=1}^{n}\mu_i\left(t_iB_i + T_i - c_ig_i\right) \qquad (2.10)$$

where μ_0 is the Lagrange multiplier on the central government's budget constraint and μ_i is the Lagrange multiplier for the subnational government i's budget constraint. The social planner maximize (2.10) with respect to the t_i, g_i, and T_i. The first-order conditions with respect to the tax rates, intergovernmental transfers, and public service provisions can be expressed as follows:

$$\mu_0 = \omega_{B_0} \cdot MCF_0 \qquad (2.11)$$

$$\mu_i = f_i \cdot \beta_i \cdot MCF_i \qquad (2.12)$$

$$\mu_i = f_i \mu_0 \qquad (2.13)$$

$$\omega_{g_0} \cdot MB_{g_0} = \mu_0 \cdot c_0 \qquad (2.14)$$

$$f_i \beta_i MB_{g_i} = \mu_i \cdot c_i \qquad (2.15)$$

where ω_{B_0} is the distributional characteristic of the central government's tax base, defined by:

$$\omega_{B_0} = \sum_{i=1}^{n}\beta_i\left(\frac{f_i B_{i0}}{B_0}\right) \qquad (2.16)$$

ω_{g_0} is the distributional characteristic of the central government's public service, defined by:

$$\omega_{g_0} = \sum_{i=1}^{n}\beta_i\left(\frac{f_i MB^i_{g_0}}{MB_{g_0}}\right) \qquad (2.17)$$

and $MB_{g_0} = \sum_{i=1}^{n} f_i MB_{g_0}^i$ is the total marginal benefit from provision of the central government's public service. The ω_{B_0} reflects the incidence of the central government's taxes across the federation, and it will be a higher if the central government's tax bases are relatively large in the subnational governments with high β values. Similarly the ω_{g_0} reflects the distributional pattern of the central government's expenditures, and it will be higher if the benefits from increased provision of the central government's public service are relatively high in subnational governments with high distributional weights.

The Lagrange multiplier in the central government's budget constraint, μ_0, can be interpreted as the social marginal cost of raising revenue by the central government, $SMCF_0$. Given this interpretation for μ_0, the first-order conditions can be interpreted as conditions for vertical and horizontal fiscal balance in a federation. *Vertical fiscal balance in taxation* is achieved because the social marginal cost of funds is equalized between the central government and any subnational government i:

$$SMCF_0 \equiv \omega_{B_0} \cdot MCF_0 = \beta_i \cdot MCF_i \equiv SMCF_i \qquad i = 1,2,\ldots n \tag{2.18}$$

This condition implies that the distributionally-weighted marginal cost of raising revenue is equalized between the two levels of government. From this, it also follows that *horizontal fiscal balance in taxation* is achieved because the distributionally-weighted cost of taxation will be the same for all subnational governments:

$$SMCF_j \equiv \beta_j \cdot MCF_j = \beta_i \cdot MCF_i \equiv SMCF_i \qquad i, j = 1,2,\ldots n \tag{2.19}$$

Turning to the first-order conditions for public spending, *vertical fiscal balance in expenditures* is achieved because the following condition will hold:

$$\frac{\beta_i MB_i}{\omega_{g_0} MB_{g_0}} = \frac{c_i}{c_0} \tag{2.20}$$

That is, the ratio of the distributionally-weighted marginal benefits of the public services provided by a subnational government and the central government equal their relative marginal costs of production. Equation (2.20) could be interpreted as the equality of the *social* marginal rate of substitution between g_0 and g_i with the marginal rate of transformation between g_0 and g_i.

Similarly, *horizontal fiscal balance in expenditures* will also be achieved because the following condition will hold:

$$\frac{\beta_i MB_i}{\beta_j MB_j} = \frac{c_i}{c_j} \tag{2.21}$$

That is, the ratio of the distributionally-weighted marginal benefits from the public services provided by any two subnational governments will equal the ratio of their marginal costs of production. Equation (2.21) could be interpreted as the equality of the *social* marginal rate of substitution between g_j and g_i and the marginal rate of transformation between g_j and g_i.

This analysis highlights the key role that unconditional transfers play in achieving vertical and horizontal fiscal balance in a federation because the optimal set of transfers equalizes the social marginal cost of public funds across all governments in the federation. Furthermore, decentralized fiscal decision-making would implement the optimal fiscal system. For example, if subnational government i receives the lump-sum transfer T_i, it would provide the level of the public good defined by the condition $MB_{g_i} = MCF_i \cdot c_i$. Multiply both sides of this condition by β_i, leads to the condition $\beta_i \cdot MB_{g_i} = \beta_i \cdot MCF_i \cdot c_i = SMCF_0 \cdot c_i$ and the conditions for vertical and horizontal fiscal balance would be achieved.

Having defined the conditions for vertical and horizontal fiscal balance, we can now provide a meaningful economic interpretation of the notion of a *fiscal imbalance*. A *vertical fiscal imbalance* occurs when the SMCFs are not equalized between the two levels of government. For example, if $SMCF_0 < SMCF_i$ for all i = 1, 2, …, n and the distributionally-weighted marginal cost of raising revenue at the federal level is less than at the subnational government level, there will be a vertical imbalance in taxation that will also be reflected in an imbalance in the spending between the two levels of government. That is, the ratio of the distributionally-weighted marginal benefits from the subnational governments' public services to the central government's services will exceed the relative costs. In this case, we will have excess taxation imposed at the subnational government level and excessive expenditure by the central government.

A *horizontal fiscal imbalance* can similarly be defined as a situation in which the SMCFs differ across subnational governments. These differentials in the social cost of raising revenue will also be reflected in an inefficient distribution of spending across the subnational governments. The ratios of the distributionally-weighted marginal benefits from the public services provided by the subnational governments will not equal their relative production costs. Thus our model provides economically meaningful definitions for the concepts of vertical and horizontal fiscal imbalance.

If the set of intergovernmental transfers is not optimal, then there will be a net social gain from changes to the level and distribution of transfers. Consider the case where initially there are no fiscal transfers and there is a vertical fiscal imbalance—the SMCFs for all the subnational governments are higher than the federal government's SMCF. The net social gain, NSG, from a one dollar per capita increase in transfers from the central government to all the subnational governments would be equal to the following:

$$NSG = \sum_{i=1}^{n} f_i \beta_i MCF_i - \omega_{B_0} MCF_0 = \sum_{i=1}^{n} f_i \beta_i \left[MCF_i - MCF_0\right] + MCF_0 \left[\sum_{i=1}^{n} f_i \beta_i - \omega_{B_0}\right] \qquad (2.22)$$

Thus the net social gain from increasing the lump-sum transfers to all the subnational governments can be decomposed into the gain the arises because MCFs of the subnational governments exceed the MCF of the central government and the gain or loss that arises if the distribution of the transfers is different from the distribution of the central government's tax base across the subnational governments.

Although our model shows that concerns for distributional equity will normally play a major role in determining the magnitude and distribution of the intergovernmental transfers, a concern for distributional equity is not a necessary condition for the existence of an optimal set of intergovernmental transfers.[4] For example, if a society placed the same distributional weight on income increases by all individuals, such that $\beta_i = 1$ for all i, then the measurement of the net social gain from a one dollar increase in transfers to all subnational governments would reduce to:

$$NSG = \sum_{i=1}^{n} f_i MCF_i - MCF_0 \qquad (2.23)$$

In other words, the net social gain would be equal to the difference between the average MCF for the subnational governments and the central government's MCF. In the absence

[4] In this we differ with Oates (1999, p. 1127) who has argued that the justification for "intergovernmental equalizing transfers" is based on "social values" not efficiency gains.

of distributional concerns, the NSG formula in (2.23) provides a measure of the extent of the vertical fiscal imbalance in a federation.

We can think of the implementation of the optimal set of unconditional intergovernmental transfers as occurring in two stages. First, we could derive a set of transfers among the subnational governments that would equalize their social marginal cost of public funds at some level, μ. We could think of this stage as the optimal fiscal equalization program. Then we could devise a set of transfers from or to the central government to equalize the social marginal cost of funds between the two levels of government. We could think of this stage as determining the optimal vertical fiscal gap for the federation. With this two stage procedure in mind, we consider the optimal equalization grants in the next section and then in Section 2.4, we consider the transfers that will determine the optimal fiscal gap.

2.2 Optimal Equalization Grants

Many federal countries, such as Australia, Canada, and Germany, provide equalization grants to their subnational governments on the basis of "deficient fiscal capacity" or "fiscal need". Equalization grants have been justified on the basis of the need to correct incentives for fiscally-induced migration, to promote fiscal equity in the federation, or to provide insurance against regional fiscal shocks. See OECD (2007). In this section, we outline the optimal taxation approach to fiscal equalization grants—one that is based on the desire to achieve horizontal fiscal balance, as defined by equations

(2.19) and (2.21), by equalizing the social marginal cost of public funds across subnational governments.[5]

We will use the same simple framework developed in the previous section to describe the optimal taxation approach to the design of an equalization transfer system. We will assume that the transfers occur between the subnational governments. In the literature, this is referred to as a net equalization system, i.e. the central government does not contribute to the equalization system.[6] The objective of the optimal equalization system is to equalize the social marginal cost of funds across all subnational governments at some level μ. Consequently, with the optimal system of equalization transfers, we will have:

$$\beta_i MCF_i = \frac{\beta_i}{1 + t_i \eta_i} = \mu \tag{2.24}$$

and the provision of the public service in subnational government i will be g_i^*. Let $E_i = c_i g_i^*$ represent the per capita expenditure of subnational government i with horizontal fiscal balance in the federation. It will be convenient to call E_i the *expenditure needs* of subnational government i. We will discuss the determinants of expenditure needs in greater detail later in this section. For the moment, we will take E_i as given and independent of the subnational government i's fiscal capacity. From the subnational government's budget constraint, $t_i = (E_i – T_i)/B_i$. In addition, we will define the subnational government's actual *fiscal capacity* as $\phi_i^* = B_i/\eta_i < 0$. Note that fiscal capacity depends not only on the size of the government's tax base, but also on the tax

[5] For a more detailed discussion of the theory of optimal equalization grants, see Dahlby and Wilson (1994).
[6] See Dahlby (2008, Chapter 10) on the ability of a gross equalization system to eliminate horizontal fiscal imbalances.

sensitivity of its tax base. The greater the tax sensitivity of the base, the lower the government's fiscal capacity because its marginal cost of public funds will be higher. Substituting these expressions in (2.24) and solving for T_i we obtain:

$$T_i = E_i + \phi_i^* - \frac{\beta_i \phi_i^*}{\mu} \tag{2.25}$$

With a net equalization system, $\sum_{j=1}^{n} f_j T_j = 0$, and therefore summing the T_i in (2.25), we can solve for μ:

$$\mu = \frac{\sum_{j=1}^{n} f_j \beta_j \phi_j^*}{\sum_{j=1}^{n} f_j E_j + \sum_{j=1}^{n} f_j \phi_j^*} = \frac{\sum_{j=1}^{n} f_j \beta_j \phi_j^*}{E + \phi^*} \tag{2.26}$$

where E is the average per capita expenditure need in the federation and ϕ^* is the average per capita fiscal capacity of the subnational governments. Note that since $\mu > 0$, $E + \phi^* < 0$. Substituting (2.26) into (2.25), we obtain the following general formula for the optimal equalization transfer to subnational government i:

$$T_i = E_i + \phi_i^* - \left(\frac{\beta_i \phi_i^*}{\sum_{j=1}^{n} f_j \beta_j \phi_j^*} \right) \left[E + \phi^* \right] \tag{2.27}$$

This equation shows that the optimal equalization transfer for each subnational government will be determined by three components. It will be increasing in the subnational government's expenditure need, E_i, decreasing with the subnational government's fiscal capacity, ϕ_i^*, and increasing its the distributionally-weighted relative fiscal capacity. The latter term is positive to reflect an increase in transfers to subnational governments that have high fiscal capacities (and therefore subnational governments that would have higher tax rates levied on them) but that also have high distributional weights, i.e. where higher tax rates impose higher distributional burdens. This third term takes equity considerations into account in the allocation of the tax burden across the subnational governments. (As we will see, equity consideration will also influence the expenditure need component E_i).

It is interesting to consider two special cases of the general formula in (2.27). First, suppose the social welfare function does not embody distributional concerns, $\beta_i = 1$ for all i. In that case, the equalization transfer is equal to:

$$T_i = \left(\frac{E_i}{E} - \frac{\phi_i^*}{\phi} \right) E \tag{2.28}$$

or the difference between the subnational government's expenditure need compared to the national average and its fiscal capacity relative to the national average multiplied by the average expenditure need. If, in addition, it is assumed that all subnational governments have the same expenditure needs and their tax bases are equally tax sensitive, i.e. $E = E_i$ and $\eta_i = \eta$ for all i, then the optimal equalization transfer is equal to:

$$T_i = \left[1 - \frac{B_i}{\sum_{j=1}^{n} f_j B_j} \right] E \tag{2.29}$$

As shown below, (2.29) is the same as the equalization grant formula under the representative tax system (RTS) equalization grants.

Under most equalization grant formulas, a subnational government that receives a transfer will be able to raise a standard amount of revenue if it imposes a standard tax rate on its tax base, or:

$$T_i + t_s B_i = t_s B_s = E \tag{2.30}$$

where T_i is the equalization transfer of subnational government i, B_i is its per capita tax base, t_s is the "standard" tax rate, B_s is the "standard" per capita tax base, and E is the per capita expenditure that the subnational government will be able to finance. The equalization transfer is therefore equal to the difference between the standard tax base and the subnational government's per capita tax base multiplied by the standard tax rate or:

$$T_i = t_s \left[B_s - B_i \right] \tag{2.31}$$

Under the RTS method of calculating equalization transfers:

$$B_s = \sum_{j=1}^{n} f_j B_j \tag{2.32}$$

$$t_s = \frac{\sum_{j=1}^{n} t_j f_j B_j}{\sum_{j=1}^{n} f_j B_j} = \frac{E}{B_s} \tag{2.33}$$

Substituting (2.32) and (2.33) into (2.31), we obtain (2.29). In other words, the RTS system of equalization grants would replicate the optimal equalization grants if distributional equity is not important, if expenditure needs are the same in all subnational governments, and if the tax bases of all of the subnational governments have the same tax sensitivity, i.e. if $\beta_i = 1$, $E_i = E$, and $\eta_i = \eta$ for all subnational governments. These are very strong assumptions, and in general the RTS equalization system, although widely viewed as the "gold standard" for equalization formulas, will only yield an approximation to the optimal equalization grants.

Our model of the optimal equalization grants emphasizes a point that has been largely ignored in the literature on equalization grants—a subnational government's fiscal capacity depends on the tax sensitivity of its tax base as well as the size of its tax base.[7] For example, suppose two subnational governments have the same per capita tax base, but one subnational government covers a large geographic area and the other a small geographic area. According to the RTS approach, the two jurisdictions would have the same fiscal capacity and be eligible for the same equalization transfers. However, the tax base of the smaller subnational government might be more tax sensitive than that of the larger subnational government because the residents on average live closer to the "border" where they can avoid paying the tax. In this way, the smaller subnational government may have a lower fiscal capacity than a larger subnational government with the same per capita tax base because its tax base is more tax sensitive. Similarly, subnational governments that are located near the "frontier" or border with another country may have more elastic tax bases than those that are located in the "interior". The RTS approach ignores differences in fiscal capacity that arise out of differences in tax base sensitivities, and therefore the RTS system may be biased against subnational governments that are smaller in size or have inherently more sensitive tax bases. Of course, measuring fiscal capacity in a way that adjusts for the tax sensitivities of tax bases is difficult given our lack of knowledge about such sensitivities. What the model shows, however, is that federations should not slavishly adhere to the RTS formula. At best, the RTS formula can be thought of as an approximation to the optimal equalization grant, and some judicious modifications to the RTS equalization grants, such as adjustments for size or location of the subnational governments, might be made to more accurately reflect the subnational governments' actual fiscal capacities.

[7] On conventional measures of fiscal capacity for the US states, see Berry and Fording (1997), Compson (2003), and Mikesell (2007).

The model of optimal equalization grants also show how equity concerns and expenditure needs should be incorporated in the computation of equalization grants. Indeed, our model provides a precise definition to the term "expenditure needs", a concept that is only vaguely defined in most of the literature on equalization grants. In our model, the expenditure needs of a subnational government are $E_i = c_i g_i^*$ where g_i^* is the public service provision level which satisfies the following condition for horizontal fiscal balance in expenditures:

$$\beta_i MB^i\left(g_i^*\right) = \mu c_i \tag{2.34}$$

where μ is the social marginal cost of funds for all subnational governments under the optimal equalization grant scheme. To discuss the expenditure needs in more detail, we will assume that the utility function of the representative household in subnational government i is $U^i = C_i + \alpha_i \ln(g_i - \gamma_i) + A_i \ln(g_0 - \Gamma_i)$ where C_i is the consumption of private goods, the α_i and A_i are positive parameters reflecting the strength of preference for local and national public services respectively, and γ_i and Γ_i are positive parameters reflecting the minimum required levels of local and national public services. It is reasonable to expect that these parameters will vary across subnational governments. Indeed, it is the variation in "tastes" for public services across subnational governments which provides one of the strongest arguments for a federal form of government.[8] Given these differences in preferences and requirements for local public services, we would expect differences in expenditure requirements, as defined in (2.34), across subnational governments. Given the utility function specified above, the expenditure needs component will be equal to:

[8] See Oates (1999 and 2005) on the rationale for adopting a federal form of governments.

$$E_i = c_i g_i^* = c_i \gamma_i + \frac{\alpha_i \beta_i}{\mu} \qquad (2.35)$$

Thus the cost of producing the basic level of service, the strength of the preference for local public services, equity considerations and the social marginal cost of public funds will jointly determine the expenditure needs of a subnational government. (Note that E_i is independent of the subnational government's fiscal capacity except in so far as its fiscal capacity affects μ, and this would only be significant if the subnational government is large relative to the entire federation.) E_i will generally be higher in subnational governments where the cost of producing public services are higher, but the elasticity of E_i with respect to c_i would be less than one because with this type of preference function, the demand for local public services is price inelastic. Specifically, $(\partial E_i / \partial c_i)(c_i / E_i) = c_i \gamma_i / E_i < 1$. Thus higher costs of producing services in a subnational government would not be fully reflected in its expenditure needs. The second component determining expenditure needs, $\alpha_i\beta_i/\mu$, will be higher if distributionally-weighted marginal benefit from providing public services is higher. For example, E_i would be higher for public transit services in an urban area with relatively low income than in a rural area with the same income level. This component of the expenditure needs will, however, be lower if the social marginal cost of funds is higher. In other words, if the social marginal cost of funds is high, the expenditure needs component will be reduced to reflect the high cost of providing funds for the equalization grant program. In other words, "affordability" as reflected in the social marginal cost of public funds will determine in part how generous the equalization program will be.

This presentation of the optimal equalization grant has ignored three important effects that equalization grants can have on the behaviour of subnational governments.

First, subnational governments will usually be faced with a formula that will determine their equalization grant and the subnational government's fiscal choice may affect the parameters of the formula and therefore the transfers that it receives. See Smart (2007). These potential incentive effects will modify the design and level of equalization payments. Second, most subnational governments levy taxes on more than one tax base and the equalization system may affect the tax rates that a subnational government imposes on its various tax bases to finance a given level of expenditures. Dahlby and Wilson (1994) show how the optimal equalization grant formula would be modified when subnational governments levy more than one tax. Third, many equalization grant systems are gross equalization systems—the central government finances the grants and subnational governments with high fiscal capacity or low expenditure needs do not contribute to the equalization fund.

2.3 The Optimal Vertical Fiscal Gap

The analysis in Section 2.1 indicated that in the absence of distributional concerns, a social planner would want to equalize the MCFs between the levels of government through a system of intergovernmental lump-sum transfers. Of course, distributional considerations are always important in determining economic policies, but it useful to set them aside to focus on the "efficiency" rationale for vertical intergovernmental transfers. The notion that intergovernmental grants can promote a vertical fiscal balance within a federation is often expressed in the literature, but this concept has not been given a strong analytical foundation.[9]

[9] However, see Boadway and Tremblay (2006).

In this section, we will focus on the role of intergovernmental transfers in achieving vertical fiscal balance in a federation. The intergovernmental transfers required to achieve horizontal fiscal balances were dealt with in the previous section. In this section, we will assume that all of the subnational governments are identical so there are no horizontal fiscal imbalances. To further economize on notation, we treat the subnational governments as a single government, denoted by a subscript 1, while the central government is denoted by a subscript 0. Furthermore, we assume that there are no vertical or horizontal fiscal externalities in the federation. (We deal with horizontal expenditure externalities in Section 3.0.) Obviously this assumption means that we will ignore, for the time being, many potentially important issues in the determination of intergovernmental grants, but we can justify this simplification because we want to isolate the issue of vertical fiscal imbalance and highlight the role of intergovernmental grants in correcting this imbalance.

It will be convenient to represent each government's tax base as follows:

$$ln\, B_i = ln\, b_i + t_i \eta_i \qquad i = 0, 1 \qquad (2.36)$$

where B_i is the per capita tax base for government i, t_i is its tax rate, $\eta_i < 0$ is the semi-elasticity of the tax base with respect to the tax rate, i.e. $\eta_i = d\, ln\, B_i / dt_i$ and b_i can be interpreted as the size of the tax base in the absence of taxation. For simplicity, we will treat η_i as a constant. A government's own-source tax revenue can be expressed as follows:

$$R_i = t_i B_i = t_i b_i e^{t_i \eta_i} = z_i \phi_i e^{z_i} \qquad \text{where} \quad z_i = t_i \eta_i \quad \text{and} \quad \phi_i = b_i / \eta_i \qquad (2.37)$$

where z_i is the proportionate rate of change in the tax base as a result of taxation and $\phi_i < 0$ can be interpreted as a measure of the fiscal capacity of a government.[10] As previously noted, a government's fiscal capacity depends not only on the size of its per capita tax base, but also on the tax sensitivity of the tax base. In other words, a government's fiscal capacity is lower if its tax base displays greater tax sensitivity. Including tax sensitivity in the measurement of fiscal capacity is extremely important in discussing vertical fiscal imbalance in a federation because the tax bases of subnational governments are almost always more tax sensitive than the central government's tax bases. Consequently, we will generally assume that $\phi_0 < \phi_1 < 0$ even if $b_0 = b_1$.

With the optimal set of taxes and intergovernmental transfers, the MCFs of the two levels of government will be equalized. This condition implies that $z = t_0\eta_0 = t_1\eta_1$ and therefore the tax bases of both levels of government are reduced in the same proportion, -z. This provides another way of defining vertical fiscal balance in federation, and another way of interpreting the *Ramsey Rule* for optimal taxation. In other words, vertical fiscal balance is achieved in a federation with the implementation of the Ramsey Rule—distortionary taxes should reduce the tax bases of the two levels of government in the same proportion. This condition can also be interpreted as implementing the *Inverse Elasticity Rule* of optimal taxation. In the context of a federation, the Inverse Elasticity Rule implies that each government's tax rate should be inversely proportional to the tax sensitivity of its tax base. The optimal intergovernmental transfers allow a federation to implement these two alternative ways of expressing the conditions for optimal tax systems.

[10] Here fiscal capacity is measured based on the size of the tax base in the absence of taxation. In the previous section it was measured based on the current level of tax; hence, the difference in the notation used for the fiscal capacity variable.

This simple model can be used to derive an expression for the (approximate) optimal fiscal gap, where we define the fiscal gap as the proportion of the subnational governments' expenditures that are financed by transfers from the central government. (Our model is very general, and the optimal transfers can be from the subnational governments to the central government under circumstances to be described below.) The following equations determine the optimal intergovernmental transfers, tax rates, and public services provided by both levels of government:

$$MB^0(g_0) = \frac{1}{1+z} \tag{2.38a}$$

$$MB^1(g_1) = \frac{1}{1+z} \tag{2.39a}$$

$$z\phi_0 e^z = g_0 + T \tag{2.40a}$$

$$z\phi_1 e^z = g_1 - T \tag{2.41a}$$

where the public services provided by the central and subnational governments, g_0 and g_1, have constant unit costs, $c_0 = c_1 = 1$, the $MB^i(.)$ functions are the marginal benefits from the public services, and T is the intergovernmental transfer. If $T > 0$, the central government transfers funds to the subnational government. If $T < 0$, the subnational government transfers funds to the central government. These four equations determine g_0, g_1, z, and T. Note that with the optimal transfers the marginal benefits from public services provided by the two levels of government are equalized because we have assumed that they have the same unit costs.

As with most optimal tax problems, it is impossible to obtain reduced form solutions for the policy variables without making some specific assumptions about functional forms. In our case, a reduced form equation for the optimal vertical transfer can be obtained by adopting the following approximations to the four equations:

$$g_0 \approx \alpha_0(1 - \varepsilon_0 z) \tag{2.38b}$$

$$g_1 \approx \alpha_1(1 - \varepsilon_1 z) \tag{2.39b}$$

$$z\phi_0(1 + z) \approx g_0 + T \tag{2.40b}$$

$$z\phi_1(1 + z) \approx g_1 - T \tag{2.41b}$$

In (2.38b) and (2.39b) we assume that the marginal benefit functions can be inverted to solve for g_i and then the g_i can be approximated using a first-order Taylor series. The α_i parameters can be interpreted as "expenditure requirements" of a government or in other words the level of service that would be provided with lump-sum taxation. The ε_i parameters can be interpreted as the "price elasticities of demand" for the public services. In (2.40b) and (2.41b) we have approximated e^z by $(1 + z)$. Summing (2.40b) and (2.41b) and substituting for g_0 and g_1 from (2.38b) and (2.39b), we obtain the following equation that determines the approximate value of z:

$$\alpha_0(1 - \varepsilon_0 z) + \alpha_1(1 - \varepsilon_1 z) = (\phi_0 + \phi_1)z(1 + z) \tag{2.42}$$

The solution for z in (2.42) is still quite complicated, and we have examined the solutions for two special cases, namely $\varepsilon_0 = \varepsilon_1 = -1$ and $\varepsilon_0 = \varepsilon_1 = 0$. In both cases, the expression for the optimal fiscal gap is the same and equal to:

$$\frac{T}{g_1} = \frac{\alpha_1\phi_0 - \alpha_0\phi_1}{\alpha_1(\phi_0 + \phi_1)} \tag{2.43}$$

From (2.43) we see that $T \gtreqless 0$ as $\dfrac{\alpha_1}{-\phi_1} \gtreqless \dfrac{\alpha_0}{-\phi_0}$. The central government should make transfers to the subnational government if the subnational government's expenditure requirement relative to its fiscal capacity exceeds the central government's expenditure requirement relative to its fiscal capacity. In essence, this condition expresses the intuition that many economists have used to justify vertical fiscal transfers. What we have done is to derive it from a model of optimal fiscal decisions, and we have provided precise definitions for the terms "expenditure requirements" and "fiscal capacity" that have been used to express these ideas.

As previously noted, we normally expect $-\phi_1 < -\phi_0$ and therefore unless the expenditure responsibilities of the central government are correspondingly higher than those of the subnational governments, we would expect transfers to go from the central government to the subnational governments. The model also predicts the intuitive results that the optimal fiscal gap will be higher when (a) the central government's expenditure responsibilities are lower, (b) the subnational governments' expenditure responsibilities are higher, (c) the central government's fiscal capacity is higher, or (d) when the subnational governments' fiscal capacities are lower.[11] The model therefore indicates how a country's expenditure assignment and tax assignment determine its optimal fiscal gap.

The following numerical example may help to put the model in perspective. Suppose $\alpha_0 = \alpha_1 = b_0 = b_1 = 100$ and that $\eta_0 = -0.25$ and $\eta_1 = -0.50$. Both levels of government have the same expenditure responsibilities and the same potential tax base. However, the subnational governments' tax base is twice as tax sensitive as the central government's tax base, and therefore the central government has twice the fiscal capacity of the subnational governments. With these parameter values, $T/g_1 = 1/3$, i.e. central

[11] Volden (2007, Proposition 3, p.220) reaches somewhat similar conclusions about the size of intergovernmental grants based on a model of bargaining model between a central and subnational government.

government transfers should cover one-third of the subnational governments' expenditures. Note for clarification that this would be a lump-sum grant which in equilibrium would be equal to one-third of subnational governments' expenditures. It would not be a matching grant of one-third of their expenditures. Note also that this measure of the optimal fiscal gap does not include conditional grants to correct fiscal externalities or fiscal equalization grants to the subnational governments from the central government.

3.0 Expenditure Externalities and Conditional Grants

Expenditure externalities arise when the activities of one jurisdiction affect the well-being of individuals in the rest of the federation, and they can distort fiscal policies because subnational governments will have biased perceptions of the total marginal benefit from their expenditures.[12] Matching grants from the central government, based on the subnational government's expenditures or revenues, can correct these biases by altering the "prices" that the subnational governments face when they make their decisions. This corrective role of matching grants has been a standard topic in the fiscal federalism literature for many years. See for example Gordon (1986) and Inman and Rubinfeld (1996). The conventional rule for determining the corrective matching rate is to set it equal to the proportion of the total marginal benefit of the activity that accrues to

[12] In this section, we are only concerned with horizontal expenditure externalities and we ignore vertical expenditure externalities and the horizontal and vertical tax externalities that can occur in a federation. On the corrective mechanisms for the latter types of externalities, see Dahlby (2008, Chapter 9).

non-resident. However, this rule has to be modified when governments rely on distortionary taxes to fund their activities.[13]

Let g_i be a publicly-provided service by subnational government i at a constant marginal cost of production of c. If the central government provides a matching grant at the rate m_{g_i} for expenditures on g_i, the optimal level of g_i from the perspective of the subnational government i will be determined by the following version of the Atkinson-Stern condition:

$$SMB_i = SMCF_i\left[\left(1 - m_{g_i}\right)c - R^i_{g_i}\right], \qquad (3.1)$$

where SMB_i is the social marginal consumption benefit that g_i provides to the residents of subnational government i, $SMCF_i$ is its social marginal cost of funds, and $R^i_{g_i} = t_i dB_i/dg_i$ is the effect of an additional unit of the public service on its revenues, where $R^i_{g_i}$ may be positive or negative. As an example, consider expenditures on a transportation infrastructure by a municipal government. SMB_i would be the social marginal benefits obtained by residents of the municipality i and $R^i_{g_i}$ would be the change in its tax revenues because lower transportation costs may increase economic activity, wages, and profits in the municipality, thereby expanding the government's tax bases. To maximize the net social gain from this type of expenditure, the central government should set the matching grant rate such that the following condition is satisfied:

$$SMB_i + SMB_j = SMCF\left[c - R^i_{g_i} - R^j_{g_i}\right], \qquad (3.2)$$

[13] The same comment applies to the simple Pigouvian tax rule to correct private sector externalities—set the tax rate equal to the marginal external damage caused by the activity. This rule is only valid when the government finances its activities with non-distortionary lump-sum taxes.

where SMB_j is the direct marginal benefit that accrue to the residents of other municipalities who use the transportation system, and $R^j_{g_i} = t_j dB_j / dg_i$ is the effect on the other municipalities revenues of an additional unit of g_i. SMB_j and $R^j_{g_i}$ could be positive or negative depending on whether they confer benefits or costs on the residents of other municipalities or increase or reduce their bases.

The above condition implies that the social marginal benefit from the activity will equal its total marginal cost, where the costs and benefits reflect distributional concerns and the use of distortionary taxes to finance spending. This condition is based on the assumption that there is a system of lump-sum intergovernmental transfers, as described in Section 2.1, that equalizes the social marginal cost of funds across the governments, such that $SMCF_i = SMCF_j = SMCF_0 = SMCF$, where $SMCF_0$ is the marginal cost of funds for the federal government. Under these conditions the optimal matching grant rate is equal to:

$$m_{g_i} = \frac{SMB_j + SMCF \cdot R^j_{g_i}}{SMB_i + SMB_j + SMCF\left(R^i_{g_i} + R^j_{g_i}\right)}. \qquad (3.3)$$

i.e. the ratio of the direct and indirect marginal expenditure externalities to the total social marginal benefit from the provision of g_i. The direct marginal benefits should be weighted according to the distributional weights that apply to the residents of the subnational governments, and the revenue effects generated by an additional unit of g_i should be weighted by the SMCF. If the marginal expenditure externalities are negative, perhaps because of adverse impacts on the environment in neighbouring jurisdictions or because the activity reduces tax revenues in other jurisdictions, then the optimal matching rate will be negative and the central government should tax g_i to discourage its provision. However, in many countries it would be difficult to enforce a measure such as this. In Canada, for example, the constitution prohibits one level of government from taxing another level of government. Under these conditions, perhaps the best that the central government can do is to seek an agreement among the subnational governments to limit the activities that generate negative expenditure externalities, or if possible change the expenditure assignment to remove the activity from the competence of the subnational governments.

Henceforth, we only consider the case where the expenditure externality is positive, i.e. $SMB_j + SMCF \cdot R^j_{g_i} > 0$, such that the optimal matching rate will be positive. In general, the effect of an increase in the SMCF on the matching rate is given below:

$$\frac{dm_{g_i}}{dSMCF} \gtreqless 0 \quad \text{as} \quad R^j_{g_i} SMB_i \gtreqless R^i_{g_i} SMB_j \tag{3.4}$$

If $R^j_{g_i} / R^i_{g_i} = SMB_j / SMB_i = \theta$, then the matching rate is not affected by SMCF, and the convention rule for the matching rate, $m_{g_i} = \theta/(1+\theta)$, would apply. A numerical example may help to clarify how the matching rate is affected by the use of distortionary taxes to finance spending. Suppose $SMB_i = 1$ and $SMB_j = 0.25$ so that 20 percent of the total direct benefits of the activity accrue to people outside the boundaries of subnational government i. The matching rate under the convention rule would be 0.20. However, suppose $R^i_{g_i} = 0.4$ and $R^j_{g_i} = 0.2$, i.e. 33 percent of the additional revenue generated by the activity accrues outside of the boundaries of subnational government i. If the SMCF is 1.5, the optimal matching rate should be 0.256. If the SMCF is 2.00, the optimal matching rate would increase to 0.265.

For a pure direct expenditure externality, where $R^j_{g_i} = 0$, the matching rate is higher when the SMCF is higher if the activity reduces the revenues of subnational government i, given that the SMB_j is positive and therefore m > 0. For a pure indirect expenditure externality, where $SMB_j = 0$, the matching rate increases with the SMCF if and only if $R^j_{g_i} SMB_i > 0$. Since we normally expect $SMB_i > 0$, this implies that the matching rate will normally be increasing with the SMCF when the activity increases the revenue of other jurisdictions. An example of an activity where SMB_i is negative, but subnational government i undertakes the activity because $R^i_{g_i}$ is large and positive, is tax enforcement. In that case, we might expect $SMB_j = 0$ but $R^j_{g_i} > 0$ as some residents of subnational government i move activities to neighbouring subnational governments to avoid paying higher taxes. Under these conditions, the matching rate should be positive to encourage tax enforcement activity, but the matching rate would be decreasing in the SMCF.

As previously noted, the formula for the optimal rate for a matching expenditure grant (3.3) is based on the assumption that a system of block grants has equalized the SMCF among all the governments in the federation. In other words, it was assumed that the federation has achieved vertical and horizontal fiscal balance as defined in Section 2.2. However, it is worthwhile to consider the optimal matching rate in a federation with fiscal imbalances, i.e. SMCFs differ between the subnational governments and/or between the central government and the subnational governments. In this situation, the matching grant rate that maximizes the net social gain from the activity would satisfy the following condition:

$$SMB_i + SMB_j + SMCF_i \cdot R^i_{g_i} + SMCF_j \cdot R^j_{g_i} = SMCF_i \cdot (1 - m) \cdot c + SMCF_i \cdot m \cdot c \qquad (3.5)$$

On the right-hand side of the above condition, the revenue effects generated by an additional unit of g_i are valued at the SMCF of subnational government i which undertakes this activity and the SMCF of the other subnational governments that receive more (or less) revenue as a result of this activity. On the left hand side, the first term represents the marginal social cost of subnational government i's share of the cost of the project and the second term represents the marginal social cost of the central government's share of the cost of the activity, where its financial contribution is valued at its $SMCF_0$. Under these conditions, the optimal matching rate will be equal to the following:

$$m_{g_i} = \frac{SMB_j + SMCF_j \cdot R^j_{g_i}}{SMB_j + SMCF_j \cdot R^j_{g_i} + \frac{SMCF_0}{SMCF_i}\left(SMB_i + SMCF_i \cdot R^i_{g_i}\right)} \qquad (3.6)$$

First note that the above formula for the optimal matching rate reduces to the formula in (3.3) if all governments have the same SMCF. When the SMCFs are not equalized, perhaps because of an inadequate unconditional grant system, the revenue effects of the activity will be weighted by the SMCFs for the respective subnational governments and the term in brackets in the denominator of (3.6), which represents net social gain to the subnational government that undertakes the activity, will be weighted by its SMCF relative SMCF of the central government. If we have a vertical fiscal imbalance in the federation, then the optimal matching rate will be higher than that given by (3.3) if the central government's SMCF is lower that the SMCF of the subnational government that undertakes the activity and vice versa. For example, using the same values for the direct and indirect marginal benefits from the activity, $SMB_i = 1$, $SMB_j = 0.25$, $R^i_{g_i} = 0.4$ and $R^j_{g_i} = 0.2$, the optimal matching rate would be 0.347 if $SMCF_0 = 1.20$ and $SMCF_i = SMCF_j = 1.80$, instead of 0.256 when SMCF is 1.50 for all governments. The reason why the matching rate is substantially higher in this example where there is a vertical fiscal imbalance is that there is a social gain from shifting more of the costs of the activity to the central government because it can finance the activity at a lower social cost. On the other hand, if there is horizontal fiscal imbalance, where the activity is undertaken by a "rich" subnational government with a low SMCF, then the matching rate will be reduced. For example, if $SMCF_i = 1.20$, $SMCF_0 = 1.5$, and $SMCF_j = 1.8$, the matching rate would be reduced to 0.248. However, the matching rate could be higher or lower if the activity is undertaken by a poor subnational government with a high SMCF while the governments that benefit from the expenditure externalities have a low SMCF. For example, if $SMCF_i = 1.80$, $SMCF_0 = 1.5$, and $SMCF_j = 1.20$, the optimal matching rate is 0.255.

The main point is that differences in the SMCFs among governments in a federation should be reflected in the matching grant formula. This does not mean that the main purpose of the conditional grant system should be to address vertical and horizontal fiscal imbalances in a federation. That task is best handled by unconditional grants, as outlined in Section 2.0, but if the unconditional grant system fails to equalize the SMCFs across all governments, then the matching grant system could incorporate elements that would address the fiscal balances as well as the expenditure externalities.

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