Adverse Selection and Risk Aversion in Capital Markets

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Abstract

We generalize the Boadway and Keen (2006) model of adverse selection in a capital market to allow for risk aversion on the part of entrepreneurs. We show that the Boadway and Keen conclusion-that adverse selection leads to excessive investment-does not necessarily hold when entrepreneurs are risk averse. We use their framework, with the additional assumption of risk aversion, to analyze the effect of policies that would reduce entrepreneurs' reliance on debt or equity financing by outside investors. We show that such policies, by exposing entrepreneurs to more down-side risk, may reduce the level of investment in risky projects, increase inequality and potentially reduce social welfare.

1 Introduction

Since the 1970s, economists have recognized that capital markets do not always conform to the model of an idealized market where individuals and firms can always borrow funds at an interest rate that accurately reflects the degree of risk posed by their investment projects. One major source of capital market "imperfection" is asymmetric information between outside investors and entrepreneurs. In many instances, entrepreneurs have better information about the probability that the investment will be successful, giving rise to a problem of adverse selection.

There is a good deal of controversy in the literature concerning the impact of adverse selection on capital markets. Two key papers in this literature—Stiglitz and Weiss (1981) and de Meza and Webb (1987)—drew conflicting conclusions regarding the implications of adverse selection for investment. Stiglitz and Weiss concluded that adverse selection reduces investments and that resource allocation can be improved by subsidizing the interest rates on loans. de Meza and Webb, using a variation of the Stiglitz and Weiss model, concluded that adverse selection results in excessive investment, in the sense that some projects with an expected return that is below the opportunity cost of capital will be financed, and that resource allocation could be improved by taxing the interest rate on loans. Whether or not adverse selection leads to excessive or inadequate private investment has potentially important implications for public policies, including tax policies. A symposium in the February 2002 issue of The Economic Journal contained a number of theoretical and empirical studies on the implications of asymmetric information for capital markets. In summarizing the debate, Cressy (2000) concluded that given our current knowledge about the performance of capital markets, the best advice that economists can give governments at this stage is "to do nothing" i.e. do not intervene by providing subsidies or imposing taxes.

Recent papers by Hellmann and Stiglitz (2000) and Boadway and Keen (2004) have extended the basic adverse selection framework by allowing entrepreneurs to finance their projects using either debt or equity from outside investors.[1] Boad-

[1]See also Fuest, Huber, and Tillessen (2003) and Fuest and Tillessen (2005) on adverse

way and Keen showed that de Meza and Webb's excessive investment result holds in an adverse selection model in which entrepreneurs and outside investors are risk-neutral.

The Boadway and Keen's excessive investment result challenges the prevailing view that capital market imperfections result in deficient investment, especially by small, start-up firms. However, their assumption that entrepreneurs are risk-neutral is troubling because, at least in the conventional view, most entrepreneurs are not able to hold diversified portfolio of assets. Their investment in their own firm represents the bulk of their wealth, and therefore they are exposed to a major source of risk if their firm should fail. Essentially, two sources of market failure are operating at the same time—adverse selection, which can lead to excessive investment in projects with negative net present values, and the entrepreneur's inability to diversify his wealth, which can lead to inadequate investment in high risk projects. Economic policies, including tax policies and policies that would increase the wealth that entrepreneurs can use to finance investments, need to be assessed within the context of a model that incorporates both distortions—adverse selection and non-diversified risk.

In Section 2, we generalize the Boadway and Keen model by allowing entrepreneurs to be risk averse. Simulations of the model in Section 3 indicate that their excessive investment result does not necessarily hold when entrepreneurs are risk averse. Total investment in risky projects declines with the degree of risk aversion, and there may a distortion in the mix of projects that are financed. Some projects with relatively low expected returns and negative net present values may be financed, while other high risk-high return projects with positive net present values will not be undertaken.

In Section 4, we use the model to analyze the effects of policies, such as those advocated by Hernando de Soto (2000), that would enable the poor in developing countries to utilize a larger fraction of their "true" wealth to finance investments by providing them with title to their homes and land. We show that such policies will have an ambiguous effect on the expected utilities of entrepreneurs because, while their interest payments on debt financing will decline, they will be exposed to greater down-side risk if their project fails. Entrepreneurs with project with a

selection in capital markets.

low probability of success or with a high return are most likely to be made worse off when more of their wealth has to be invested in a risky project. Consequently, investment in risky projects may decline when entrepreneurs can use a larger fraction of their wealth to finance risky investments and social welfare may or may not increase. In Section 5, we consider the effects on social welfare when entrepreneurs are wealthy enough that they do not have to rely on outside investors in the "imperfect capital market". We argue that because of the adverse selection problem, these wealthy entrepreneurs will not use debt financing, and they will not have access to the equity market. If they self-finance their investments, they will be exposed to down-side risk and simulations from our model indicate that the level of investment in risky projects may decline, compared to the level that would prevail with if entrepreneurs had to access the "imperfect" capital market. The only entrepreneurs who might be better off when self-financing is possible are those with projects with very high expected rates of return. Thus policies that reduce reliance on "imperfect capital markets", where interest rates are high because of the adverse selection problem, may reduce investment in risky projects, increase inequality, and reduce social welfare.

Finally, in Section 6 we discuss the possibility that risk averse entrepreneurs could sell more equity in their firms than the minimum required to finance their projects for risk sharing reasons. We show that, even if the returns on investments are verifiable, an informationally consistent equilibrium in the terminology of Spence (1973, 1976) may not exist if extreme punishments are not available. Indeed, agents with very poor projects may want to write contracts that they know they cannot fulfill in some states precluding fully revealing contracts for the very best projects.

2 A Model of Adverse Selection with Risk Averse Entrepreneurs

Each entrepreneur has a project that requires one unit of wealth if it is to be realized. Projects are characterized by their probability of success, p, and the magnitude of their return, R, if they are successful. If the project is unsuccessful, the return is zero. It will be assumed that $0 < \alpha < p \leq 1$ and that the returns on the individual projects are uncorrelated. Let r be the real rate of return on a safe asset. Entrepreneurs know the (p, R) characteristics of their own projects. Outside investors only know the distribution of p and R across all projects, but they are aware that the owners of the firm have private information about the probability of success and the magnitude of payoff from the project if it is successful.

The entrepreneur's wealth, W, consists of two types of assets, $W = L + K$. One asset, L, cannot be used to finance investment in the project. For example, L could be the earning of a spouse or an asset that cannot be pledged as security for a loan because of legal restrictions or the absence of full property rights. The other asset, K, can be used to finance investment in the risky project, but we assume in this section that $K < 1$ and therefore the entrepreneur requires outside financing in order to make the investment in his project. Let $\varphi = 1 - K$ be the level of outside investment that is required to finance the project.

Debt Financing First consider debt financing. The interest rate on debt financing for these projects is i. Competition among lending institutions implies that the market interest rate on debt is determined by the following condition:

$$\hat{p} \equiv \mathbb{E}\left[p|\text{debt financing}\right] = \frac{1+r}{1+i}, \tag{1}$$

where $\hat{p}$ is the average probability of success (and therefore repayment of a loan) of a debt-financed project and r is the risk-free rate of return that the lending institutions pay to their depositors. It is assumed that r is fixed and that the lending institutions can hold a well-diversified portfolio of loans so that they are treated as risk neutral.

If the entrepreneur borrows to finance the project and it is successful, he pays off the loan and his wealth is $(R - (1+i)\varphi)$. It is assumed that entrepreneurs have a constant relative risk aversion utility function,

$$u\left(c\right) = \frac{c^{1-\sigma}}{1-\sigma} \text{ for } \sigma \neq 1 \text{ and } u\left(c\right) = \log c \text{ for } \sigma = 1$$

If the entrepreneur uses debt to finance his project, his expected utility is

$$EU_{DF} = p\frac{\left(R + L - \left(1+i\right)\varphi\right)^{1-\sigma}}{1-\sigma} + \left(1-p\right)\frac{L^{1-\sigma}}{1-\sigma},$$

where σ is the coefficient of relative risk aversion, $0 \leq \sigma$, $\sigma \neq 1$. For $\sigma = 1$, the utility function is the log of wealth and the expected utility with debt finance is

$$EU_{DF} = p \log \left(R + L - (1+i)\,\varphi\right) + (1-p) \log L.$$

If the entrepreneur invests in the safe asset, he will have a secure amount of wealth, W_s, which is defined below:

$$W_s = L + (1+r)\,(1-\varphi)$$

Entrepreneurs will prefer debt financing their projects to investing K in the safe asset if

$$R \geq Z\,(p) - L + (1-i)\,\varphi, \tag{2}$$

where

$$Z\,(p) = L \left[\frac{(W_s/L)^{1-\sigma} - (1-p)}{p}\right]^{\frac{1}{1-\sigma}} \quad \text{for } \sigma \neq 1,$$

and

$$Z\,(p) = L \left(\frac{W_s}{L}\right)^{1/p} \quad \text{for } \sigma = 1.$$

$Z(p)$ can be interpreted as the level of wealth that the individual needs if the project is a success in order to make the expected utility with the investment equal to the expected utility from investing in the safe asset. Note that Z is decreasing in p and increasing in σ. Note also for Z to be positive the range of p values has to be restricted as follows:

$$p > \alpha \equiv 1 - \left(\frac{L}{W_s}\right)^{\sigma-1}.$$

If the entrepreneur is risk neutral, $Z(p,0) = (1+r)(1-\varphi)p - 1 + L$ and (4) can be written as:

$$R \geq \frac{(1+r)\,(1-\varphi)}{p} + (1+i)\,\varphi \tag{3}$$

In Figure 1, which shows the Boadway and Keen equilibrium with risk neutral entrepreneurs, the BB' curve represents the projects with (p, R) values such that risk-neutral entrepreneurs are indifferent between investing in their project using borrowed funds and investing their wealth in the safe asset.

Equity Financing Now consider equity financing by outside investors. Suppose that the total number of shares issued in a firm is normalized to equal one, and the price of a share is the stock market value of the firm, V. The outside equity investors will receive (φ/V) of the profits if it is successful and 0 if it is failure. Investors bid up firm's share prices until:

$$V = \frac{\mathbb{E}\left[pR|\text{equity financing}\right]}{1+r} \tag{4}$$

In other words, the value of a firm is equal to the present value of its expected return, given that it accepts equity financing, discounted at the risk free rate of return.

The entrepreneur's expected utility with equity financing is the following:

$$EU_{EF} = p\frac{1}{1-\sigma}\left(R\left(1-\frac{\varphi}{V}\right)+L\right)^{1-\sigma} + (1-p)\frac{L^{1-\sigma}}{1-\sigma},\ \sigma \neq 1, \tag{5}$$

and

$$EU_{EF} = p\log\left(R\left(1-\frac{\varphi}{V}\right)+L\right) + (1-p)\log L,\ \sigma = 1, \tag{6}$$

Hence, entrepreneurs would rather finance their projects with equity than invest in the safe asset if

$$R \geq \frac{V}{V-\varphi}\left[Z\left(p\right)-L\right]. \tag{7}$$

If the entrepreneur is risk neutral, (7) can be written

$$R = \frac{V}{V-\varphi}\frac{\left(1+r\right)\left(1-\varphi\right)}{p} \tag{8}$$

The EE' curve in Figure 1, shows combination of (p, R) values such that the owners of the firm are indifferent between equity financing and investing their wealth in the safe asset.

The Choice Between Equity and Debt Financing Firms that would accept equity financing may prefer debt financing. The entrepreneur's degree of risk aversion does not affect the preferred method of financing, and debt financing is preferred to equity debt financing if $pR(1-(\varphi/V)) < p(R-(1+i)\varphi)$. Entrepreneurs are indifferent between debt and equity financing if

$$R = \left(1+i\right)V. \tag{9}$$

That is, entrepreneurs are indifferent between debt and equity financing if the stock market value of the firm is equal to the present value of the return on the investment discount at the interest rate on debt. The curve MM' in Figure 1 represents the projects where the owners are indifferent between debt and bond financing. A firm with a project whose R lies below MM' will prefer equity financing to debt financing.

The curves intersect at the same point which is given by:

$$p_{MEB} = \frac{L^{1-\sigma} - {W_s}^{1-\sigma}}{L^{1-\sigma} - \left[(V-\varphi)(1+i)+L\right]^{1-\sigma}} \quad \text{for } \sigma \neq 1 \tag{10}$$

$$p_{MEB} = \frac{\log W_s - \log L}{\log\left[(V-\varphi)(1+i)+L\right] - \log L} \quad \text{for } \sigma = 1 \tag{11}$$

Note that

$$p_{MEB} = \frac{1-\varphi}{V-\varphi}\hat{p}, \tag{12}$$

when $\sigma = 0$.

Therefore entrepreneurs with projects whose (p, R) values that lie above the line segment BJM' in Figure 1 will finance their projects with debt. These projects with relatively high R values are debt-financed so that the owners do not have to share the high returns with outside shareholders. Entrepreneurs with projects in the region $JM'E'$ will be financed in the stock market. Equity financing is attractive for the owners of firms with projects that have relatively low R values and relative high probabilities of success.

The projects where the expected rate of return is greater than or equal to the rate of return on the safe asset satisfy the condition $pR > (1+r)$. In Figure 1, all of the projects that satisfy this condition lie on or above the FF' line. These are the projects that would be financed under symmetric information by risk neutral investors. The FF' line intersects the MM' line at the probability level:

$$p_{MF} = \frac{\hat{p}}{V} \tag{13}$$

where $p_{MEB} < p_{MF}$. Therefore point A in Figure 1 lies to the right of point J.

Market Equilibrium Equilibrium in the market is determined by zero profit conditions, (1) and (4), and the self-selection conditions determined by (2), (7), (9) and (11).

Figure 1 illustrates Boadway and Keen's over-investment result with risk neutral entrepreneurs. The debt-financed projects in the region $DJAF$ will have a negative expected net present value because the FF' and the BB' curves interest at the probability level $\hat{p}$ and BB' is steeper than FF'. Also note that the EE' curve is always below the FF' curve and in particular that since V is greater than one in equilibrium. (V can be interpreted as Tobin's average q.) The equity-financed projects in the region $JE'F'A$ will have negative expected present values. Thus, the Boadway and Keen model predicts that all of the projects with positive expected net present values will be financed and that some projects with negative expected net present values will be also be financed either by debt or equity.

If entrepreneurs are risk averse, it is possible that some projects with a positive net present value will not be financed. Intuitively, as the degree of risk aversion increases, holding V and i constant, p_{MEB} increases, shifting point J in Figure 1 towards point K. This reduces the region of over-investment. Note, however, that as the degree of risk aversion changes and poor projects are dropped, the equilibrium values for i and V will also change. Our intuition suggests that i will decrease and V will increase as risk aversion increases, which would have offsetting and therefore ambiguous effects on the position of the J and K. Therefore a general equilibrium analysis is required to assess the full effect of an increase in risk aversion on the level of over-investment. Furthermore, the slopes of the BB' and EE' curves will also change when entrepreneurs become more risk averse, creating the possibility that the BB' curve may intersect the FF' curve above the MM' line and therefore some projects with low probabilities of success, but high R values and positive net present values, will not be undertaken by entrepreneurs because they are too risky.

Given the complexity of the model, it has not been possible to derive the conditions under which under-investment in projects with positive net present values occurs. However, in the next section we use numerical simulations to illustrate the effects of risk aversion on the market equilibrium.

3 Equilibria with Excessive and Inadequate Investment: Simulation Results

The model was simulated with the following parameter values: $r = 0.05$, $\varphi = 0.90$, $L = 0.05$, $W_s = 0.155$, $L/W_s = 0.323$. The distribution function is $f(p, R) = \theta e^{-\theta R}$ where $\theta = 1.25$. With this distribution function, $p(R > 1) = 0.287$, $p(R > 2) = 0.082$, $\mathbb{E}[p] = 0.5$, $\mathbb{E}[R] = 0.80$, $\mathbb{E}[pR] = 0.40$. The area above the FF' curve, which measures the proportion of the projects with positive net present values evaluated at the risk free rate of return, is 0.095.

Table 1 shows the computed values of the key endogenous variables with three different levels of risk aversion. In the first column, for comparative purposes, we report the equilibrium values of the variables if entrepreneurs are risk neutral. In this equilibrium, the interest rate on debt is 68.3 percent. (One of the reasons why the interest rate is so high in this model is that we have assumed that the investment has no scrap value if it fails.) The market value of the shares in the firms that are financed by equity investment is 1.03. (Recall that this can be interpreted as Tobin's average q.) The probability that a debt financed firm will repay its debt, $\hat{p}$, is 0.624. In other words, 47.6 percent of the debt-financed firms default on their loans. This also explains the high interest rate on debt in this model. Figure 2a shows the MM', BB' and EE' curves intersect at the 0.479 probability level. As noted in the previous sections, excessive investment is a characteristic of the equilibrium. In this equilibrium, 9.5 percent of all projects are debt financed and 6.6 percent are financed by equity. The total percentage of projects financed, 16.1 percent, greatly exceeds the 9.5 percent of the projects that have positive net present values.

In the second column shows, where the coefficient of relative risk aversion is 0.90, the interest rate on debt drops to 46.0 percent because the probability that a debt-financed firm will repay its debt increases to 0.719. The market value of the shares in the firms that are financed by equity investment increases to 1.034. As Figure 2b shows the MM', BB' and EE' curves now intersect at a much higher probability level, 0.696. This equilibrium is characterized by both excessive investment and under investment. (The FF' curve lies above the BB' curve in the range of p values from 0.22 to 0.69 and below the BB' curve from

$p < 0.22$.) In other words, there is under-investment in some high risk-high return projects that have positive net present values because entrepreneurs are unable to fully diversify their wealth. In this equilibrium, 9.1 percent of all projects are debt-financed and 3.4 percent are financed by equity. The total number of projects financed, 12.5 percent, exceeds the level that would be financed under symmetric information.

In the third column, where the coefficient of relative risk aversion is 2.00, the interest rate on debt drops to 20.0 percent because the probability that a debt-financed firm will repay its debt increases to 0.875. The market value of the shares in the firms that are financed by equity investment falls increases to 1.019. As Figure 2c shows the MM', BB' and EE' curves now intersect at a much higher probability level, 0.915. Now the equilibrium is characterized by under investment. The FF' curve lies almost completely below the BB' curve indicating significant under-investment in projects with positive net present values. Only 5.6 percent of all projects are debt financed and the equity market virtually evaporates. Only 0.4 percent of projects are financed by equity. The aggregate level of investment is below the level that would occur under symmetric information and the loss caused by adverse selection and the inability to pool risks, increases to 45.8 percent.

These simulations show that the excessive investment result does not necessarily hold in a capital market with adverse selection and risk averse entrepreneurs. Our numerical simulations indicate that the number of projects that are financed can either exceed, or fall short of, the number that would be financed in a perfect capital market. What the model reveals is a distortion in the mix of projects that are financed–some projects with low expected returns are financed and some high risk-high return projects with high positive net present values are not financed.

Bonds: Assume that agents can choose the amount of financing that they get at the equilibrium i. If one borrows $b > 1 - K$ one must repay $b(1+i)$ thus having an income $R - b(1+i) + L$ if successful and L if unsuccesful (assumption: if the individual defaults on his loan then the bank may seize the assets up to L.) The point here is that risk sharing can only occur with variations in i.

Equity:

4 Increasing the Proportion of Wealth that can be used to Finance Investments

An increase in K, with an offsetting reduction in L, has two effects on the incentive to invest in the risky project. First, the opportunity cost of investing in the risky project increases by the rdK, and therefore some entrepreneurs with marginal projects will find investing in the safe asset more attractive. Second, entrepreneurs will not have to borrow as much and their interest payments will decline, but at the same time they will face greater down-side risk because of the reduction in L. Consequently, the expected utility of entrepreneurs with "good" projects may either increase or decrease. To see this, we can write the expected utility of an entrepreneur with a debt-financed project as $EU = pU(R + L - (1 + i)\varphi) + (1 - p)U(L)$. The effect on expected utility of an increase in K, holding total wealth constant, is given by the following condition.

$$\left.\frac{dEU}{dK}\right|_W \lesseqgtr 0 \text{ as } i - \varphi \frac{di}{dK} \lesseqgtr \frac{1-p}{p} \frac{U'(W_1)}{U'(W_2)}, \tag{14}$$

where di/dK is the change in the equilibrium interest rate on loans to entrepreneurs and $i - \varphi di/dK$ is the reduction in the entrepreneur's interest payments from a small increase in K and an offsetting reduction in L, which shifts wealth from the loss state to the success state of the world. An entrepreneur's marginal rate of wealth substitution between the two states of the world is given by the right hand side of the second set of conditions in (14). An entrepreneur will be better off if the reduction in his interest payments exceeds his marginal rate of substitution of wealth between the two states of the world. For a given value of R, the entrepreneurs with low p projects have a higher marginal rate of substitution of wealth between the two states of the world and are more likely to be worse off when K increases and L declines. Therefore, entrepreneurs with low p projects are the ones that are most likely to drop their projects. Consequently, the default rate on loans will decrease, and the interest rate on loans will to decline, i.e. $di/dK < 0$. Holding p constant, the entrepreneurs with projects with higher R

values will have a higher marginal rate of wealth substitution, and therefore they are more likely to be made worse off. Thus, somewhat paradoxically, reduced reliance on outside financing can make some entrepreneurs with "good" projects worse off. Finally, the greater the degree of risk aversion, the higher marginal rate of wealth substitution, the more likely that an increase in K will make the entrepreneurs worse off.

As we have seen, an increase in the amount of wealth that can be used to finance investment will likely make some individuals better off and other worse off, and therefore to evaluate the distributional effects of a policy which increases K and reduces L, we will use the following social welfare function:

$$SWF = \int_p \int_R (1-\zeta)^{-1} \mathbb{E}\left[U(W)\right]^{1-\zeta} f(p,R)\, dpdR$$

where ζ is the coefficient of inequality aversion. If $\zeta = 0$, then the social welfare function is utilitarian. Any level of social welfare can be expressed in terms of the equivalent equally distributed equivalent (EDE) level of wealth and this will be our measure of social welfare.

As demonstrated in our simulations in Table 2, the level of investment may decline when K increases and L declines because entrepreneurs will be exposed to more risk when they are responsible for financing a higher proportion of the investment, and the opportunity cost of investing in the risky project may increase. In Case I, the entrepreneurs' total wealth is 1.2 but $L = 0.8$ and $K = 0.4$ and therefore they have to rely on outside investors for 60 percent of the financing of the project. Assuming $\sigma = 0.90$, $r = 0.05$, and $\theta = 1.25$, the equilibrium interest rate on loans would be 0.333, the value of a firm would be 1.069, and 10.96 percent of the projects would be financed. In Case I, with $\zeta = 0$, the EDE wealth level is 1.257436. To put this figure in perspective, note that those entrepreneurs that invest in the safe asset and who have the lowest expected utility have secure wealth equal 1.22.

Case II shows the effect of reducing L from 0.8 to 0.4 and simultaneously increasing K from 0.4 to 0.8. One could think of this increase in the amount of wealth that can be used to finance investments as arising from an increasing property rights in informal housing in a developing country. Note that the entrepreneurs would still require outside financing for 20 percent of the projects cost.

In the new equilibrium, the interest rate on loans would decline to 0.192 from 0.333 because the default rate on loans would decline by almost 10 percentage points, i.e. would increase from 0.788 to 0.881. The decline in the default rate is reflected in the reduction in investment. In Case II, only 6.96 percent of the projects would be financed. Investment in the projects declines because the entrepreneurs' secure level of wealth will have increased to 1.24 and therefore some projects with "marginal" expected returns will no longer be financed because the opportunity cost of investing in the risky projects has increased. In addition, the some entrepreneurs with "good" projects will now face a reduction in their expected utility from investing in their risky projects.

While the expected utilities of some of the entrepreneurs with good projects decline in Case II, overall social welfare increases with a utilitarian or pro-poor social welfare functions because some of the "losers" were initially better off than the "winners".

5 Self-Financing Risky Investments

{question: are we changing the nature of choices in this section? I guess the equivalent question in the previous section would be to ask what would happen if agents could choose the amount of resources they would want to use to finance their projects} To this point, it has been assumed that entrepreneurs need outside funding for their projects. Although an individual's wealth, $W = L + K$, was not restricted to be less than one, it was assumed that $K < 1$. Therefore, a fraction of the project's cost $\varphi = 1 - K$ had to be financed by debt or equity from outside investors. We will now consider the case where $K > 1$ and the individuals can finance the project from their own wealth.

We begin by showing that an entrepreneur with $K > 1$ will not finance part of his investment with debt. Then we will argue that there will be no equity investment by outside investors if outside investors can observe the wealth of the entrepreneur and the amount of equity that the entrepreneur retains in the project. Therefore, any investment in the project will be completely financed by the entrepreneur out of his own wealth. We will refer to this as self-financing.

It will never be in the interest of an entrepreneur with $K > 1$ to finance some or all of the project with debt if i, the interest rate on debt, exceeds r, the interest rate that he can earn on a secure investment because debt financing will reduce the entrepreneur's wealth in both states of the world. If I is the fraction of the cost of the project that is financed by debt, $\varphi \leq I \leq 1$ the entrepreneur's wealth if the project succeeds is:

$$\begin{aligned} W_2 &= R + L + (1+r)(K - (1-I)) - (1+i)I \\ &= R + L + (1+r)(K-1) - (i-r)I \end{aligned}$$

If the project fails, his wealth is:

$$\begin{aligned} W_1 &= L + (1+r)(K - (1-I)) - (1+i)I \\ &= L + (1+r)(K-1) - (i-r)I \end{aligned}$$

Note that we are assuming that the loan has to be fully repaid if the project fails—the entrepreneur cannot default on the loan if $K > 1$. Since W_1 and W_2 are both decreasing in I if $i > r$, the entrepreneur would never borrow to finance the project given $K > 1$. It might be argued that since the loan is risk free, $i = r$. However, the interest rate on loans has to cover some administration costs, although these are not included in our model, and therefore we predict the entrepreneur will not finance his project with debt.

An alternative method of finance is the sale of equity in the project to outside investors. Let I be the amount invested by outside equity investors where $\varphi \leq I \leq V$. If $\varphi = I$, the entrepreneur would retain no equity in the project. With equity financing, the entrepreneur's wealth in the two states of the world would be:

$$W_1 = L + (1+r)(K - (1-I)) = L + (1+r)(K-1) + (1+r)I$$

$$W_2 = L + (1+r)(K - (1-I)) + R(1 - (I/V)) = W_1 + R(1 - (I/V))$$

If the expected utility of an entrepreneur with equity financing is represented as

$$EU_{EF} = pU(W_2) + (1-p)U(W_1),$$

then equity investment by outsiders has the following effect on the entrepreneur's expected utility:

$$\begin{aligned} \frac{\partial EU_{EF}}{\partial I} &= (1+r)\left[pU'(W_2)+(1-p)U'(W_1)\right]-U'\left(W_2\right)\frac{pR}{V} \\ &= (1+r)\left[\mathbb{E}\left[U'\left(\tilde{W}\right)\right]-U'\left(W_2\right)\frac{pR}{\mathbb{E}\left[pR|EF,I\right]}\right] \end{aligned}$$

where $\mathbb{E}\left[U'\left(\tilde{W}\right)\right]$ is the expected marginal utility of income and we have substituted the equilibrium condition for the equity market, $(1+r)V = \mathbb{E}\left[pR|EF,I\right]$ where $\mathbb{E}\left[pR|EF,I\right]$ is the average expected return on equity financed investments where I is the amount of outside investment. First consider the entrepreneurs that would like to sell all of the equity in their projects, $I = V$. The above condition would be written as:

$$\frac{\partial EU_{EF}}{\partial I} = (1+r)\,U'(W_2)\left[1-\frac{pR}{\mathbb{E}\left[pR|EF,I=V\right]}\right]$$

In order for this condition to be satisfied, $pR \leq \mathbb{E}\left[pR|EF,I=V\right]$. In other words, each entrepreneur's project would have to have an expected value that is less than or equal to the average expected value for equity financed projects with no retention of ownership by the entrepreneur. Obviously, this is a contradiction and therefore entrepreneurs cannot sell all of the equity in their projects. However, an interior solution, with $\varphi \leq I < V$ means that the following condition has to hold:

$$\frac{\mathbb{E}\left[U'\left(\tilde{W}\right)\right]}{U'\left(W_2\right)} = \frac{pR}{\mathbb{E}\left[pR|EF,I\right]} > 1.$$

Each entrepreneur who sells I equity in his project would have to have an expected return that is greater than the average expected return for all equity financed projects with that level of outside investment. Again, this is a contradiction. Therefore there is no equilibrium in which an entrepreneur can finance his investment by selling shares to outsiders if he has enough wealth to self-finance. Adverse selection implies that entrepreneurs with enough wealth to finance the investment cannot access the equity market because outside investors are assumed to be able to observe the wealth of the entrepreneur and the ownership that he retains in the project.

Therefore we conclude that if an entrepreneur with $K > 1$ is going to finance a project, it will be self-financed. The condition for self-financing is:

$$p\frac{W_2^{1-\sigma}}{1-\sigma} + (1-p)\,\frac{W_1^{1-\sigma}}{1-\sigma} \geq \frac{W_s^{1-\sigma}}{1-\sigma} \qquad \sigma \neq 1$$

$$p \log W_2 + (1-p)\log W_1 \geq \log W_s \qquad \sigma = 1$$

where $W_s = L + (1+r)K$ is the entrepreneurs' secure level of wealth if he invests in the safe asset,

$$W_1 = L + (1+r)(K-1) = W_s - (1+r)$$

is wealth if the project fails, and

$$W_2 = L + R + (1+r)(K-1) = W_s + R - (1+r)$$

is wealth if the project succeeds. Note that a risk averse entrepreneur will never self-finance a project if $R < (1+r)$. The following condition indicates the value of R that is required for an entrepreneur to be indifferent between self-financing his project and investing in the risk-free asset:

$$R = \left\{ \left[\frac{(W_s/W_1)^{1-\sigma} - (1-p)}{p} \right]^{\frac{1}{1-\sigma}} - 1 \right\} W_1$$

$$R = \left[(W_s/W_1)^{1/p} - 1 \right] W_1$$

These conditions will be represented by the SS' curves in (p, R) space in Figure 3. Note that in the above conditions if $p = 1$, $R = (1+r)$ and also that if $\sigma = 0$, $R = (1+r)/p$. In other words, if the entrepreneur is risk neutral, the SS' curve would coincide with the FF' curve. With risk averse entrepreneurs, the curve SS' will lie above the FF' curve. Since FF' the curve may lie above the relevant sections of the BB' and EE' curves (if the entrepreneurs are only moderately risk averse), this implies that a self-financing may result in less investment than would occur if the entrepreneurs have to rely on debt or equity financing for their projects.

Case III in Table 2 illustrates the effect of self-financing on the level of investment and social welfare. The wealth of the entrepreneurs is the same as in the previous cases, but they can now self-finance the project. Only 5.24 percent of the risky projects are financed, in part because W_s increases to 1.26, but also because of the increase in the downside risk that the entrepreneurs face now that they finance the entire project from their own wealth. However, there is an overall social welfare gain because of the increase in W_s to 1.26.

Converting L into K may be costly and the conversion rate may be less than one for one as is assumed in Cases II and III. Case IV shows what would happen if in converting L into K, the W_s remained constant so that there was no gain to those who only invest in the safe asset. The entrepreneurs can self-finance their investment because $K = 1.162$. The computations show that only 5.01 percent of the risky projects would be financed, even though the opportunity cost of the investment remains the same as in Case I, because of the greater downside risk that the entrepreneurs face. The computed values of EDE wealth also decline, indicating a social welfare loss from . Figure 3 shows the relevant sections of the BB' and EE' curves for Cases I and the SS' curve for Case IV. In the latter only entrepreneurs only invest in the projects above the SS' curve. All of the entrepreneurs with projects between the SS' curve and the relevant sections of the BB' and the EE' curves, or 5.95 percent of the entrepreneurs are worse off in Case IV compared to Case I. The entrepreneurs with projects above the SS' curve but to the left of the curve labeled GG would also be worse off. Thus the these computations show that only entrepreneurs with projects with high (p, R) values would be better in Case IV than in Case I.

6 On the possibility of alternative financial schemes

Back to the case in which $K < 1$, it should be noted that risk averse entrepreneurs might want to sell more than equity in their firms than the minimum required to finance their projects for risk sharing reasons.

This raises the question of whether alternative equilibria exist in which entrepreneurs sell more than φ in order to have a more diversified portfolio. In this section, we consider whether an equilibrium with this characteristic could be maintained in the market in view of the pressures caused by adverse selection. Note that it cannot be optimal for an agent to finance with debt more than the minimum required, φ, whenever $i > r$. So we focus on this possibility for equity.

Although ex-ante entrepreneurs differ in two dimensions of non-observed characteristics, R is 'ex-post' verifiable when the project is successful. This allows R to be contracted upon, if we assume that strong enough punishments are available for those who have lied when the contract was signed. If p, too, were observed, at the resulting equilibrium, each entrepreneur with a project such that $pR > (1+r)$ would sell all of his equity in his project and have $L+pR-(1+r)\varphi$ with certainty. With non-observed p, for each, R one may consider a competitive signaling model in the spirit of Leland and Pyle (1977), where the share of the project an agent retains signals its quality and allows for a perfectly revealing equilibrium.

If however, 'extreme punishments' are not available, it may become worthwhile for some agents to lie about R and hope that the success state is not realized.[2] As a result, a market in the sale of equity based on R may unravel. We consider this possibility below.

Let $I(p, R)$ be the total amount of funds injected in a project of type (p, R), and let $V(p, R)$ be the projects corresponding value in a fully separating equilibrium.[3] Total shares are, thus $\alpha(p, R) = I(p, R)/V(p, R)$.

The payoffs associated with the truth-telling strategy are

$$W_2^{TT} = R\left(1 - \frac{I(p, R)}{V(p, R)}\right) + L + (1+r)\left[I(p, R) - \varphi\right]$$

in the success state and

$$W_1^{TT} = L + (1+r)\left[I(p, R) - \varphi\right]$$

in the failure state.

A false report $(\hat{p}, \hat{R})$, on the other hand, yields

$$W_2^F = L$$

in the success state and

$$W_1^F = L + (1+r)\left[I\left(\hat{p}, \hat{R}\right) - \varphi\right]$$

[2] The equilibrium of such model may not be easy to characterize, so we leave it for future work.

[3] This is an interesting benchmark case emphasized by Spence (1973, 1976) and shown to be the unique reactive equilibrium by Riley (1979).

in the failure state.[4]

An agent will lie if there is a type $(\hat{p},\hat{R})$ with $I(\hat{p},\hat{R})$ contract contract is such that

$$\begin{aligned} & pU(L)+(1-p)U\left(L+(1+r)\left[I(\hat{p},\hat{R})-\varphi\right]\right)> \\ & pU\left(R\left(1-\frac{I(p,R)}{V(p,R)}\right)+L+(1+r)\left[I(p,R)-\varphi\right]\right) \\ & +(1-p)U\left(L+(1+r)\left[I(p,R)-\varphi\right]\right) \end{aligned}$$

The value of the firm must be increasing in p and R in a fully separating equilibrium.

It is then the case that if an agent with project (p,R) lies,

$$\left(1,\bar{R}\right)=\arg\sup_{\hat{p},\hat{R}} pU(L)+(1-p)U\left(L+(1+r)\left[I(\hat{p},\hat{R})-\varphi\right]\right).$$

Hence, it will be optimal for the owner of project (p,R) to lie if

$$\begin{aligned} & pU(L)+(1-p)U\left(L+(1+r)\left[I\left(1,\bar{R}\right)-\varphi\right]\right)> \\ & pU\left(R\left(1-\frac{I(p,R)}{V(p,R)}\right)+L+(1+r)\left[I(p,R)-\varphi\right]\right) \\ & +(1-p)U\left(L+(1+r)\left[I(p,R)-\varphi\right]\right). \end{aligned}$$

The zero profit condition requires

$$\frac{pR}{1+r}=V(p,R).$$

Because, $I(p,R)\geq 1-K>0$, for $p=0$, $V(p,R)=0$. Hence, there will always be a p close to 0 for whom it is worth lying, thus breaking the fully separating equilibrium. That is, for every R there is a $p^*(R)$ such that

$$p^*(R)U\left(W_2^F\right)+(1-p^*(R))U\left(W_1^F\right)=p^*(R)U\left(W_2^{TT}\right)+(1-p^*(R))U\left(W_1^{TT}\right).$$

It is not hard to see that $dp^*(R)/dR<0$, which defines two regions, the northeast of Figure 4, in which truthful announcement is optimal and the southwest in which a false announcement is best.

[4]The argument developed henceforth go through if we assume that agents with very low quality projects are excluded from the market in the candidate separating equilibrium.

For an atomless distribution of types, the existence of such a region of false announcements excludes a separating contract in the extreme northeast of the truthful region.

Whether any region of fully separating equilibrium is possible requires further investigation of this non-standard screening problem. We have concentrated on a very specific type of partially separating equilibrium in which agents only finance the fixed amount $1 - K$, thus ruling out the possibility of using equity financing to share risk. We do so not only for tractability, but also to facilitate the communication with previous work in the literature. In future work it is our hope to have a tighter characterization of equilibria of this non-standard mechanism design problem.

7 Conclusion

We have shown that the Boadway and Keen's result—that adverse selection in a capital market leads to excessive investment—does not necessarily hold if entrepreneurs' are risk averse. However, we feel that the Boadway and Keen framework is potentially very useful for analyzing policies that effect entrepreneurial incentives to investment, and we have tried to show how the framework can be used to evaluate a policy of the wealth that entrepreneurs can use to finance investments. Other policy issues, such as the tax treatment of losses, could also be usefully analyzed using the Boadway and Keen model and incorporating the assumption of risk averse entrepreneurs.

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A Tables and Figures

Table 1 Simulations of the Capital Market Equilibrium

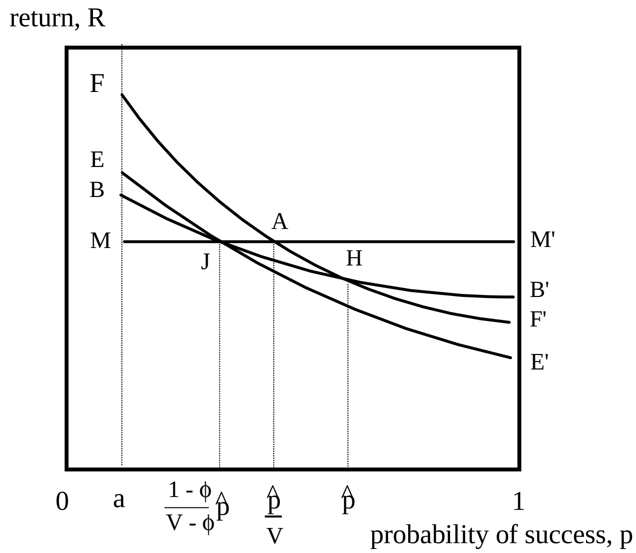
Degree of Risk Aversion	$\sigma = 0$	$\sigma = 0.9$	$\sigma = 2.0$
i	0.683	0.460	0.200
V	1.03	1.034	1.019
$\hat{p}$	0.624	0.719	0.875
p_{MEB}	0.479	0.696	0.915
Percentage of projects financed by:			
Debt	9.5%	9.1%	5.6%
Equity	6.6%	3.4%	0.4%
Debt and Equity	16.1%	12.5%	6.00%

Table 2 The Effects of Increasing Wealth that can be used to Finance Risky Investments

Case	I	II	III	IV
Key Parameter Values	$L = 0.8$	$L = 0.4$	$L = 0$	$L = 0$
	$K = 0.4$	$K = 0.8$	$K = 1.2$	$K = 1.162$
	$\varphi = 0.6$	$\varphi = 0.2$	$\varphi = 0$	$\varphi = 0$
	W_s= 1.22	W_s= 1.24	W_s= 1.26	W_s= 1.22
i	0.333	0.192	na	na
V	1.069	1.092	na	na
$\hat{p}$	0.788	0.881	na	na
p_{MEB}	0.725	0.865	na	na
Pctg. of Projects Financed				
Debt	9.05	6.41	na	na
Equity	1.91	0.55	na	na
Total	10.96	6.96	5.24	5.01
Equally dist. equivalent wealth				
$\zeta = 0$	1.257436	1.267892	1.281950	1.24084
$\zeta = 0.5$	1.256973	1.267523	1.281655	1.240553
$\zeta = 1.5$	1.256077	1.266811	1.281083	1.239980

Other parameter values: $\sigma = 0.90$, $r = 0.05$, $f\,(R) = 1.25e^{-1.25R}$

Figure 1: The Capital Markets with Adverse Selection and Risk Neutral Entrepreneurs



Notes: Debt-financed projects have (p, R) values in the area above BJM'. Equity-financed projects are in the area $JE'M'$. Projects with positive net present values lie above FF'. $\hat{p} = (1+r)\,/\,(1+i)$ represents the success rate for debt-financed projects.

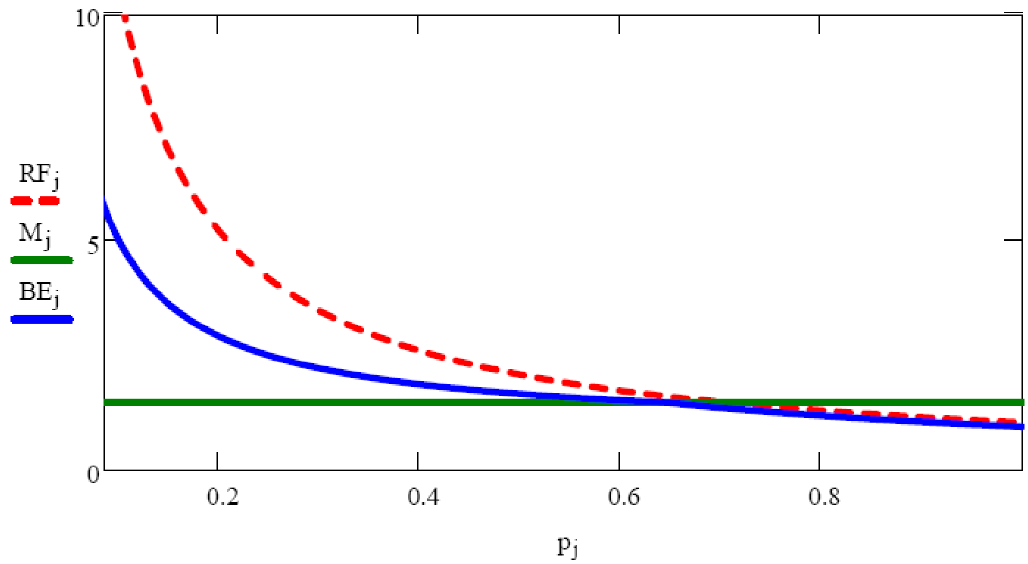


Figure 2a: Equilibrium with Risk Neutral Entrepreneurs ($\sigma = 0$)

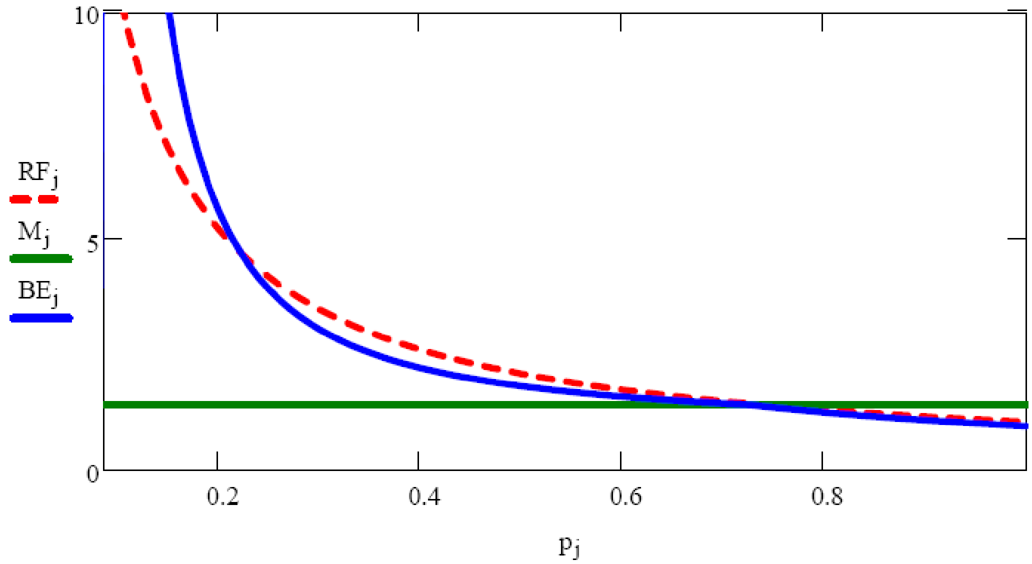


Figure 2b: Equilibrium with Risk Averse Entrepreneurs ($\sigma = 0.90$)

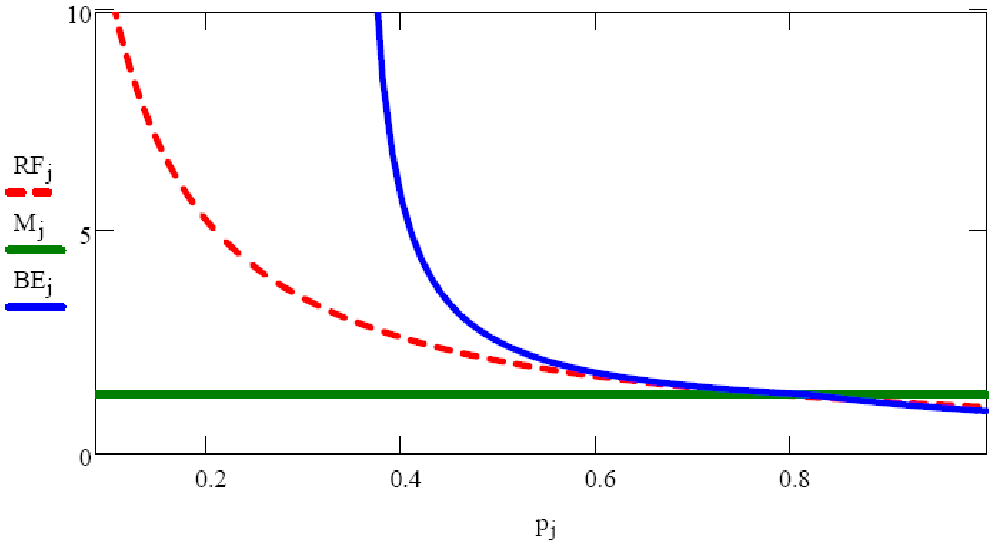


Figure 2c: Equilibrium with Risk Averse Entrepreneurs ($\sigma = 2.00$)

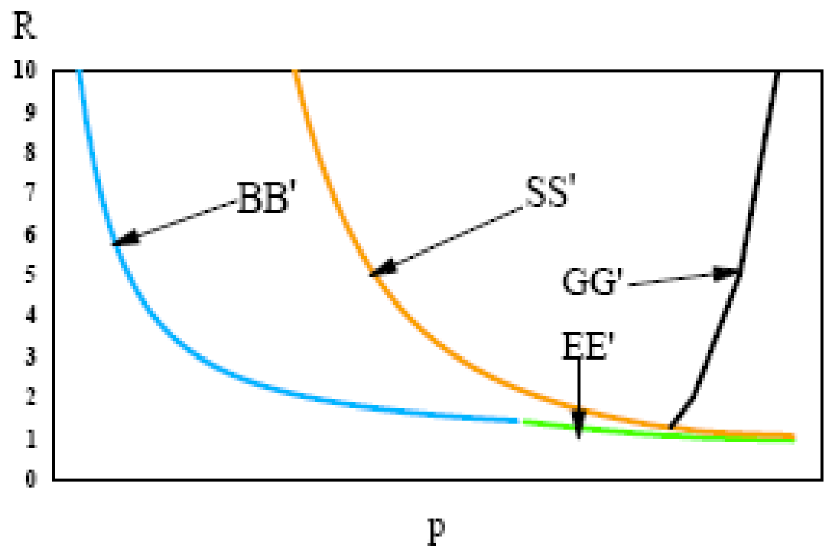


Figure 3: The Effects of Increasing Wealth Used to Finance Risky Investments